Stable Laser Resonator Modeling: Mesh Parameter Determination and Empty Cavity Modeling

Justin Mansell, Steve Coy, Kavita Chand, Steve Rose, Morris Maynard, and Liyang Xu

MZA Associates Corporation

Outline

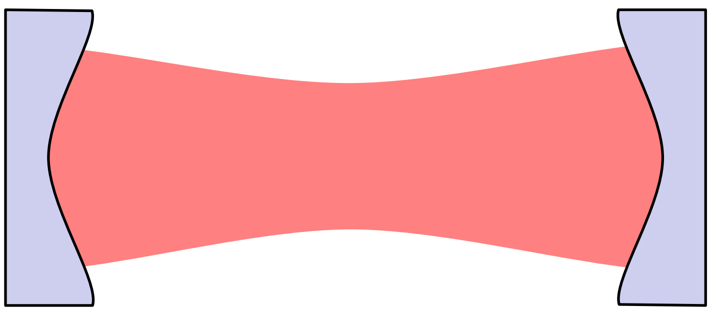
- Introduction & Motivation
- Choosing a Wave-Optics Mesh
  - Multi-Iteration Imaging
- Stable Resonator Modeling
- Conclusions

Introduction and Motivation

- When evaluating a new laser gain medium, it is common to build a multi-mode stable resonator around the gain medium to demonstrate maximum extraction.
- Modeling multi-mode lasers is difficult because
  - The modes tend to interact with the saturable gain medium and create modeling instabilities and
  - The mesh requirements for a multi-mode stable resonator are very resource intensive.

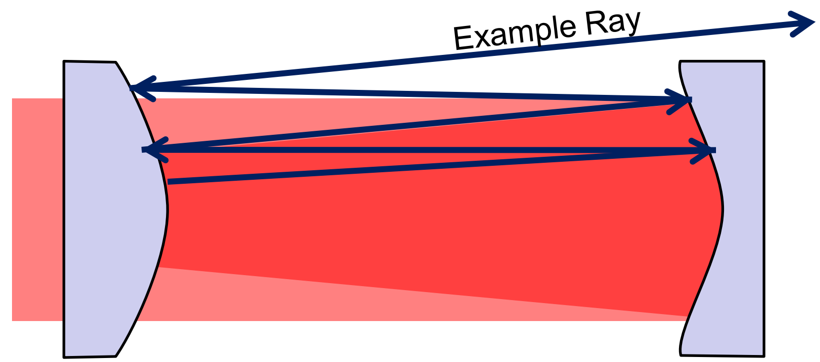
Laser Resonator Architectures: Stable vs. Unstable

Stable



Rays captured by a stable resonator will never escape geometrically.

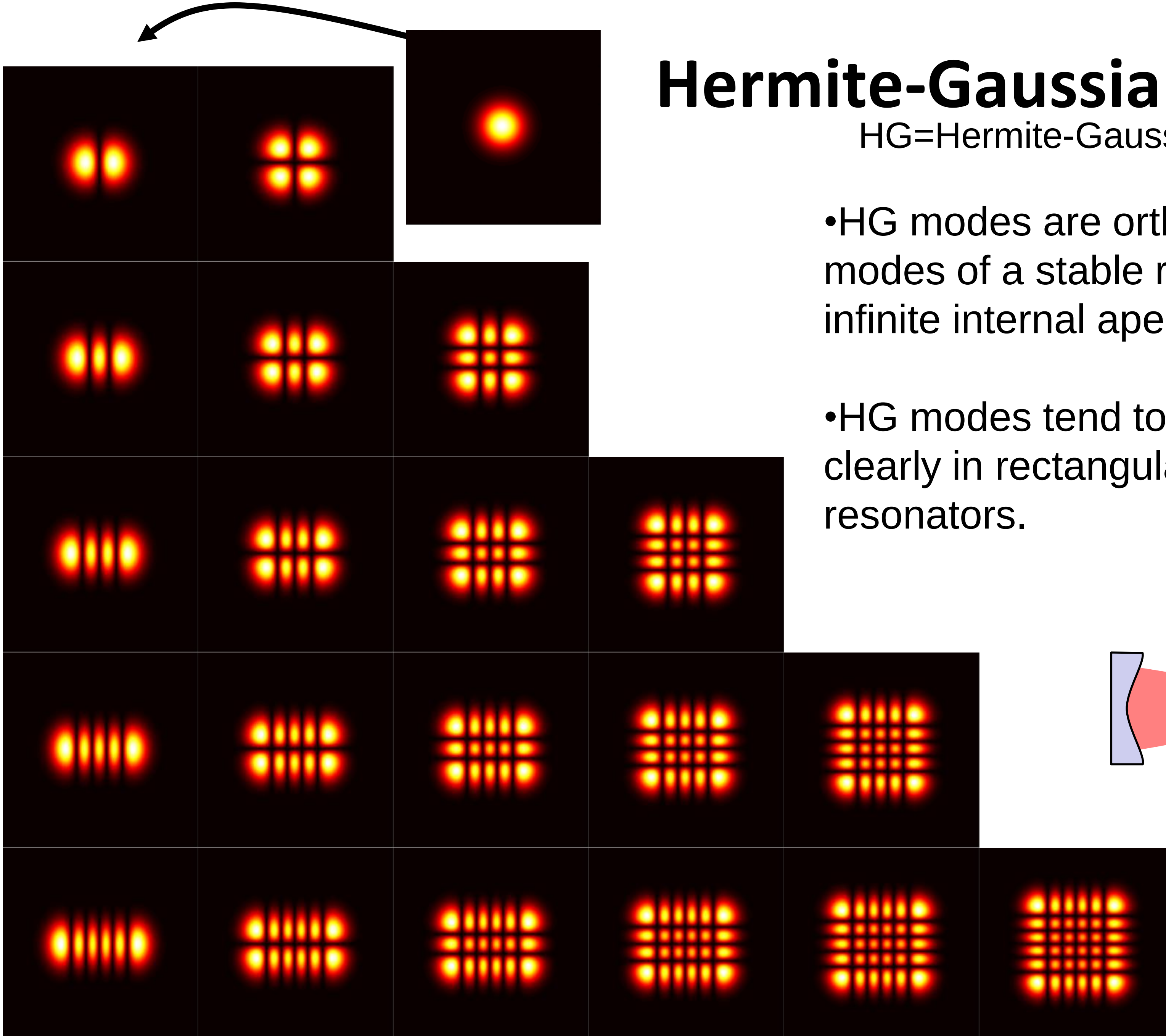
Unstable



All rays launched in an unstable resonator (except the on-axis ray) will eventually escape from the resonator.

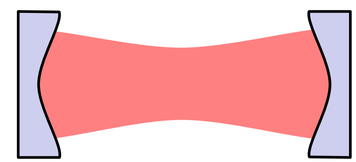
Hermite-Gaussian Modes

HG=Hermite-Gaussian



•HG modes are orthogonal modes of a stable resonator with infinite internal apertures.

•HG modes tend to appear clearly in rectangular-aperture resonators.



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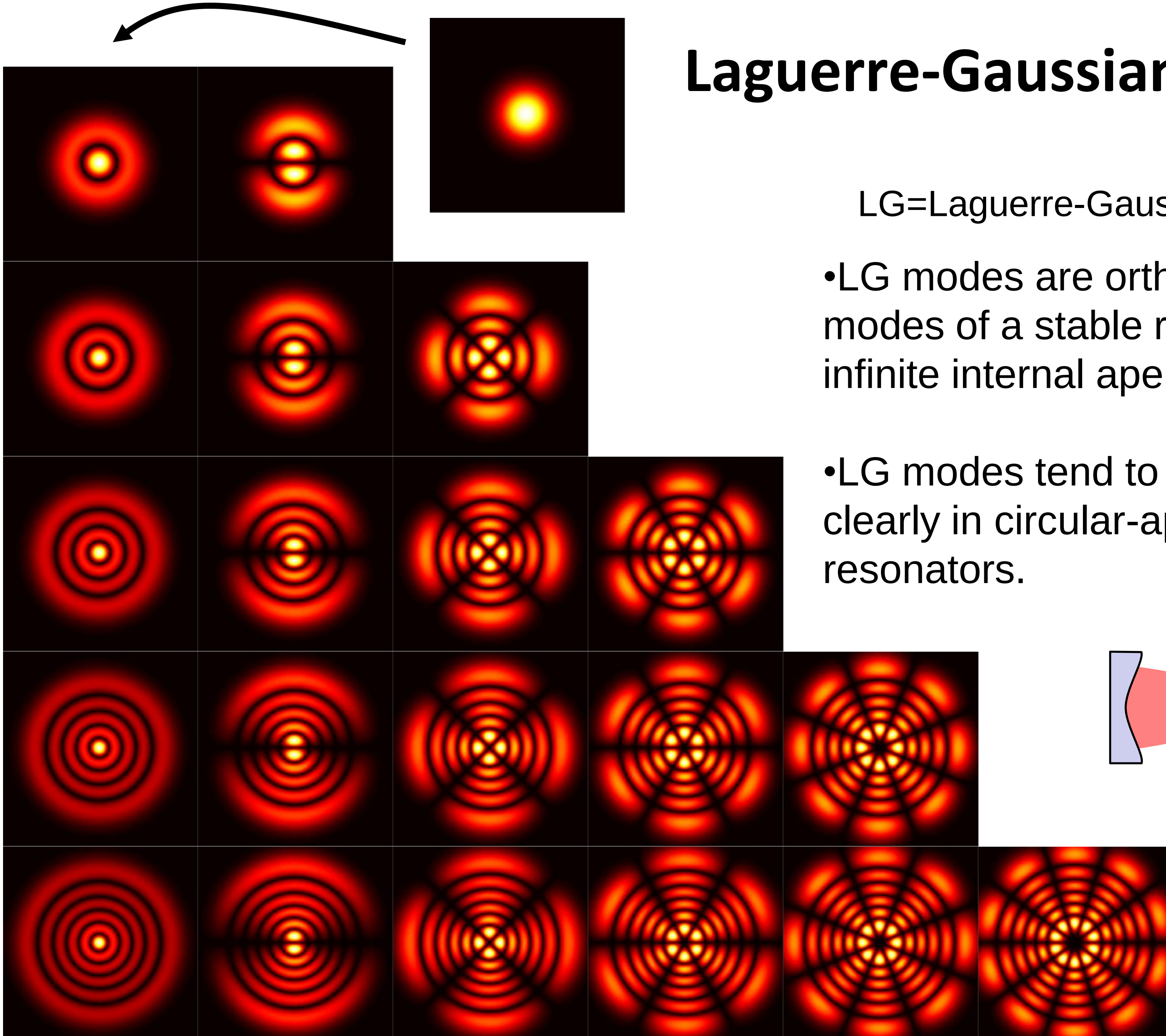
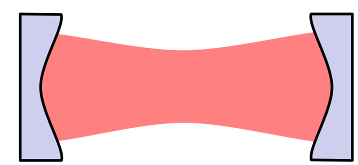


Laguerre-Gaussian Modes

LG=Laguerre-Gaussian

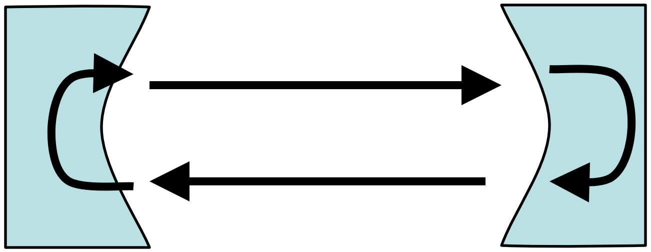
- LG modes are orthogonal modes of a stable resonator with infinite internal apertures.

- LG modes tend to appear clearly in circular-aperture resonators.



The Iterative Fourier Transform (aka Fox & Li) Technique

- A field is propagated through repeated round-trips until the field has converged to a stable field distribution.
- This is a commonly used technique for simplifying the 3D solution of Maxwell's Equations into a 2D problem (i.e. Gerchberg-Saxon technique)



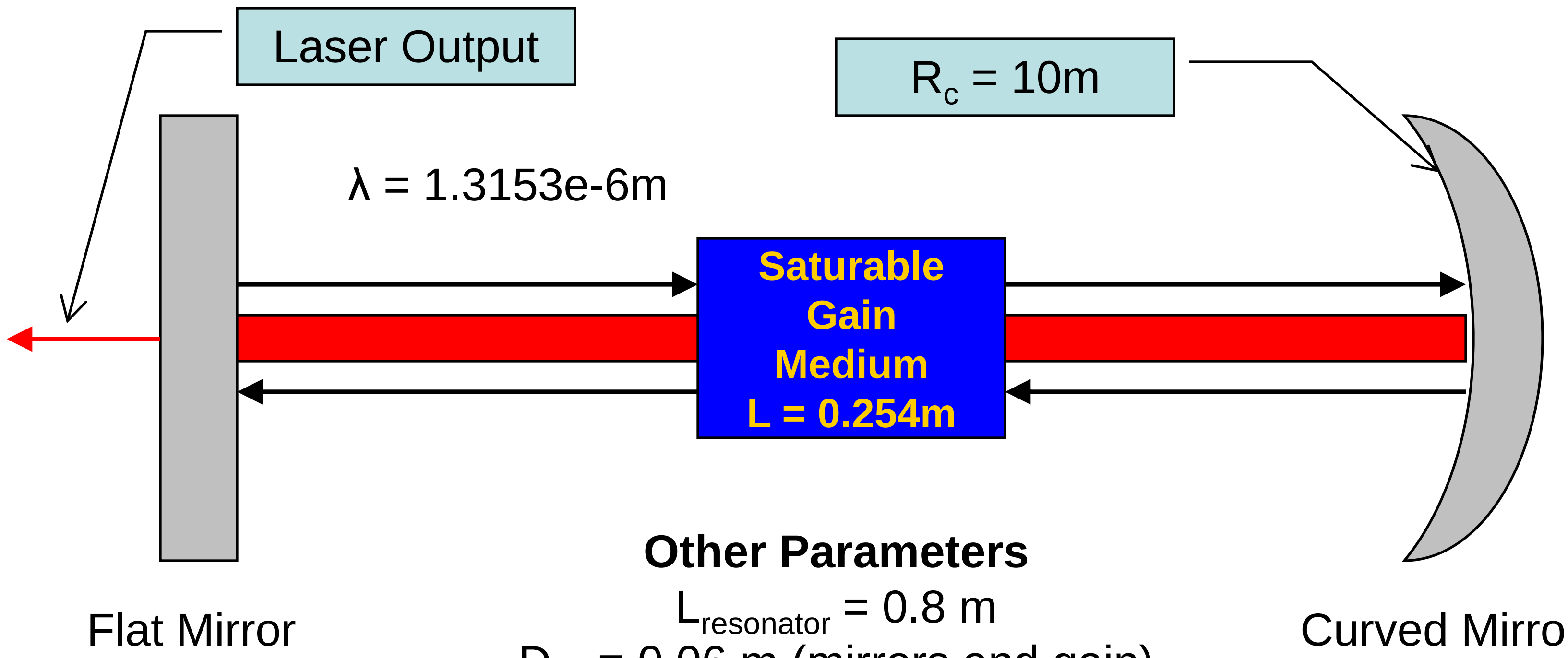
A. G. Fox and T. Li. "Resonant modes in a maser interferometer", Bell Sys. Tech. J. 40, 453-58 (March 1961).

A. G. Fox and T. Li, "Computation of optical resonator modes by the method of resonance excitation", IEEE J. Quantum Electronics. QE-4, 460-65 (July 1968).

Comments on Fox & Li Solutions

- Solution is for an instantaneous state, which is typically the steady-state of the laser.
- Stable resonators are much more computationally intensive to model than unstable resonators.
  - We attribute this to the geometric output coupling of an unstable resonator.
  - Larger eigenvalue difference between fundamental mode and the next higher-order mode.
- Not generally appropriate for pulsed or time-varying solutions unless the time-varying nature is much slower than a resonator round-trip time.
  - This is analogous to a split-time modeling techniques.

Example Resonator: RADICL



Other Parameters

$L_{resonator}$ = 0.8 m

D_{ap} = 0.06 m (mirrors and gain)

g_{ss} = 0.19435 %/cm

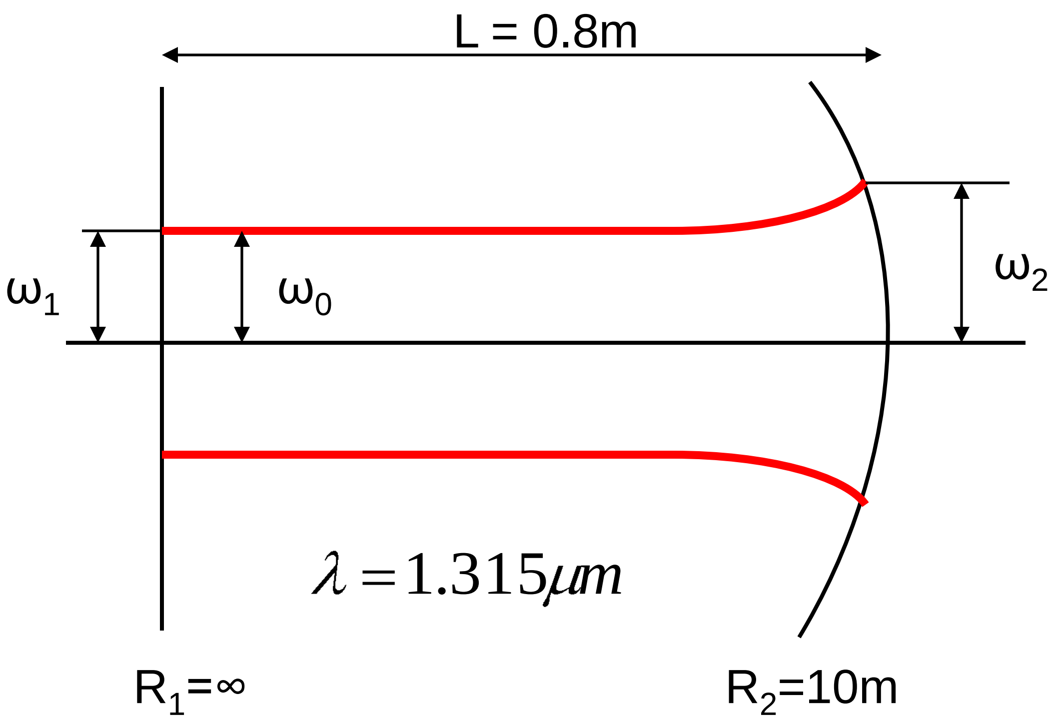
I_{sat} = 6.735 kW/cm^2

Data source:

Eppard, M., McGrory, W., and Applebaum, M. "The Effects of Water-Vapor Condensation and Surface Catalysis on COIL Performance", AIAA Paper No. 2002-2132, May 2002.

Predicted Gaussian Radius

TEM00 Mode Radius for Plano Concave Resonator



$$\omega_1^2 = \omega_0^2 = \frac{\lambda}{\pi}\sqrt{L(R_2 - L)}$$

$$\omega_2^2 = \frac{\lambda}{\pi} R_2 \sqrt{\frac{L}{R_2 - L}}$$

$$\omega_0 = \omega_1 = 1.0656mm$$

$$\omega_2 = 1.11mm$$

Wave-Optics Mesh Determination

Calculation 1: Half-Round-Trip

- Estimate based on two limiting apertures analysis [Coy/Mansell]:

$$N \geq \frac{4D_1 D_2}{\lambda \Delta z} = \frac{4 \cdot 6\text{cm} \cdot 6\text{cm}}{1.315\mu\text{m} \cdot 80\text{cm}} \cong 13{,}688, \qquad N^2 \geq 1.87\text{E}8$$

NOTE: This is for a half of a round trip.

$$\delta \leq 8.8\text{x}10^{-6}\, m$$

- 13,688 points per dimension

Very Difficult

Calculation 2: Full Round-Trip

- Mesh points = 16 * Fresnel number (Mansell, SPIE 2007)

$$N_f = \frac{a^2}{L\lambda}$$

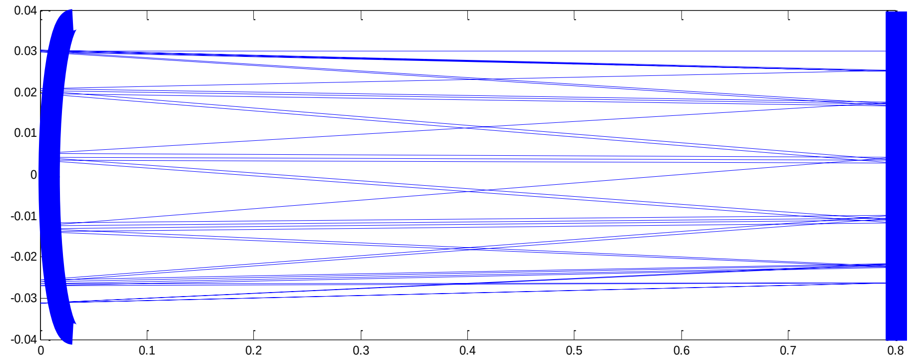
  – a = half limiting aperture size = 3 cm
  – λ = 1.315 µm
  – L = 1.7524 m = (L + (L*R/2)/(R/2-L)) = (0.8 + 0.9524) m
  – N_f = 391 for the 6 cm aperture,
    - 16*N_f = **6256** mesh points in each dimension

Still Difficult

Calculation 3: Maximum Angular Content

Ray Optics Analysis of a Stable Laser Resonator (1-D)

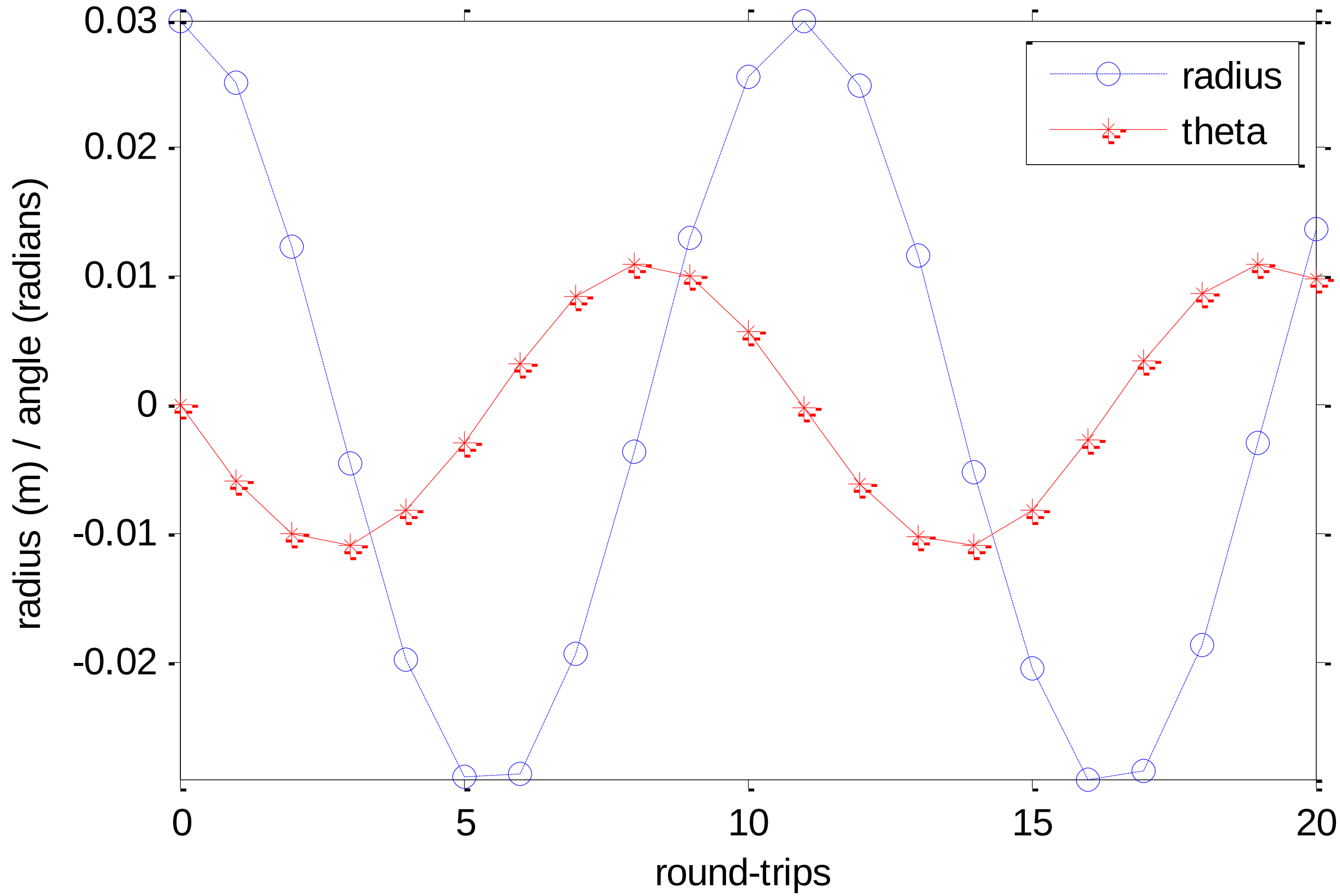
A ray launched parallel to the optical axis will oscillate back and forth across the resonator cavity.



The rays slope increases until it crosses the center of the cavity, then decreases. This gives us an estimate of θ_{max}.

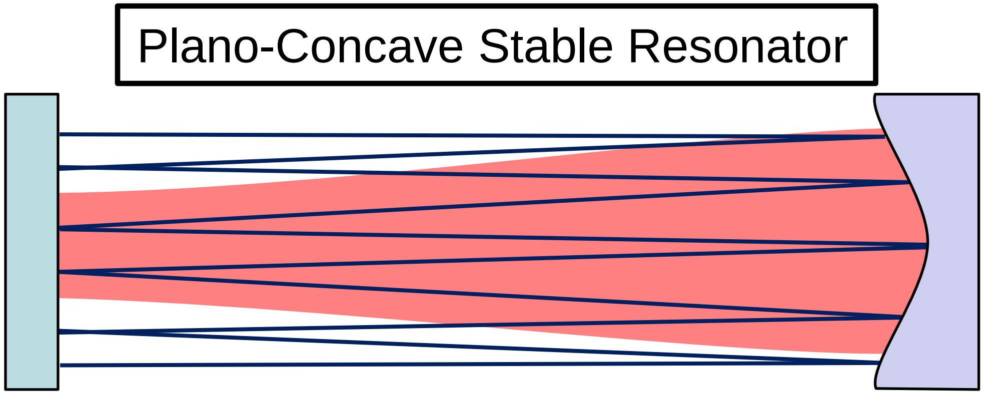
- Using ray optics we estimate θ_{max}~= 0.0111 for this resonator.
  - Obtained with ray tracing with a ray at the edge of the aperture.

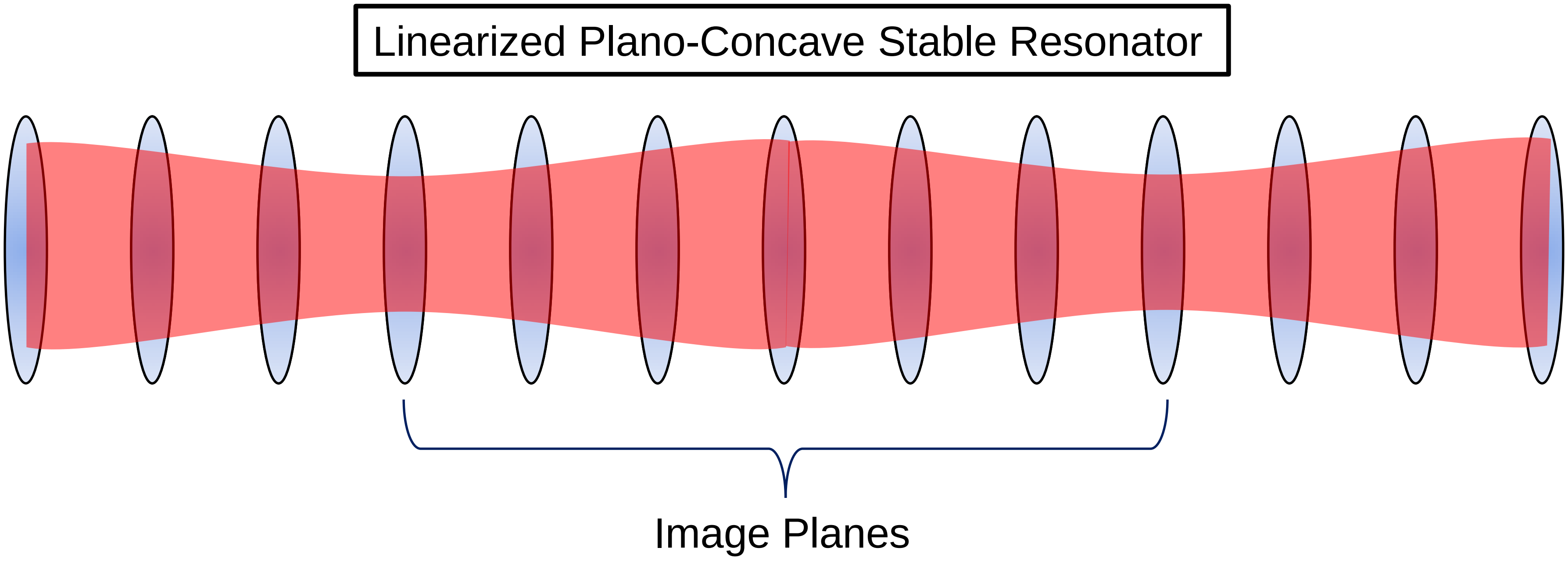
Rays and Angles Unwrapped



Graphical Method of Determining the Number of Round-Trips to Image

Number of Round-Trips to Image/Repeat





Calculation 3: Maximum Angular Content (2)

- Now that we have, we can immediately obtain the constraint on the mesh spacing by applying the Nyquist criterion:

$$\delta \le \frac{\lambda}{2\theta_{\max}} = \frac{1.315\mu\text{m}}{2 \cdot 0.0111} = 59.5\mu\text{m}$$

- Finally, we can obtain the constraint on N by imposing the requirement that no rays leaving the cavity from one side should be able to “wrap-around” and re-enter the cavity from the other side:

$$N \ge \frac{D + \theta_{\max} \cdot \Delta z}{\delta} \cong \frac{6\text{cm} + 0.0111 \cdot 80cm}{59.5\mu\text{m}} \cong 1158$$

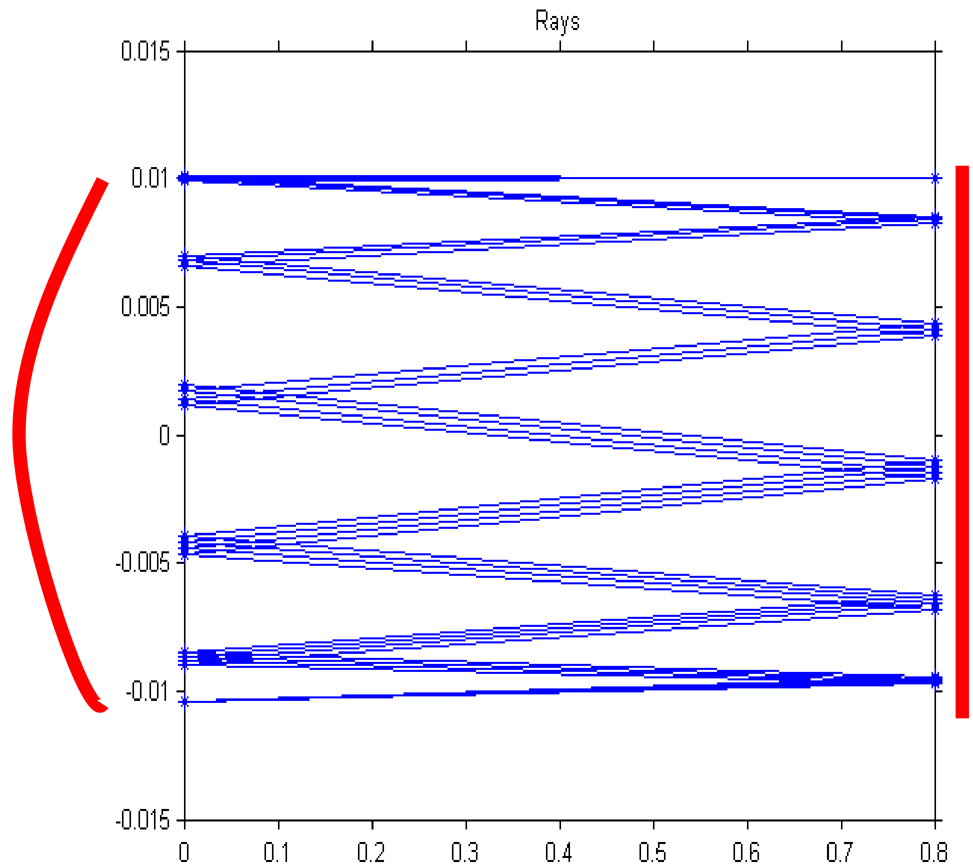
- This method makes the greatest use of resonator information, so we believe it to well-represent the minimum mesh size required.

NOTE: 13,688 / 1158 ~ 11 = number of round-trips to image

Examining the Ray Tracing

- As is typical of a stable resonator, a ray launched at the edge of the resonator walks to the other side of the resonator and then back to the starting side.
- For this resonator, this process takes ~11 round-trips
- Therefore, the angle required is approximately reduced by a factor of 11.

How can we find the number of round trips to image?



Analytical Method to Determine the Number of Round-Trips to Image

This problem can be addressed using ray matrices. Consider a resonator with a round-trip ray-matrix given by M. In N round-trips, the ray-matrix will be given by M^N. To reproduce any input ray, we need to determine the number of round trips to make the identity matrix, or M^N=I.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^N = M^N = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

This can be determined numerically, but can also be addressed using eigenvalue analysis. The approach on the right derives an equation for N assuming a plano-concave resonator with length L and the end mirror radius of curvature equal to 2f.

$$M^N v_i = \lambda^N v_i$$

$$M^N = \lambda^N$$

$$M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1-L/f & 2L-L^2/f \\ -1/f & 1-L/f \end{bmatrix}$$

$$X = 1 - L/f$$

$$\lambda = X \pm \sqrt{1-X^2}\, j$$

$$\lambda = e^{j\phi}$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-X^2}}{X}\right)$$

$$\lambda^N = e^{jN\phi}$$

$$N = \frac{2k\pi}{\phi} = \frac{2k\pi}{\tan^{-1}\left(\frac{\sqrt{1-X^2}}{X}\right)}$$

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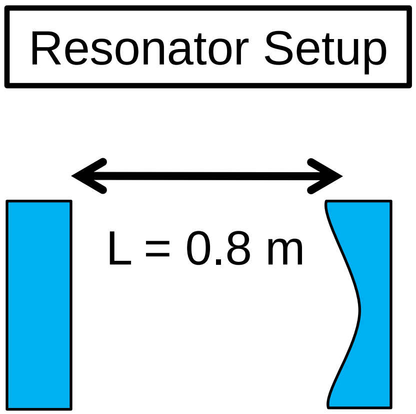
Example of Analytical Method



R = 10 m
f_{eff} = 5 m

$$M = \begin{bmatrix} 1 - L/f & 2L - L^2/f \\ -1/f & 1 - L/f \end{bmatrix} = \begin{bmatrix} 0.84 & 1.472 \\ -0.2 & 0.84 \end{bmatrix}$$

$$X = 1 - L/f = 0.84$$

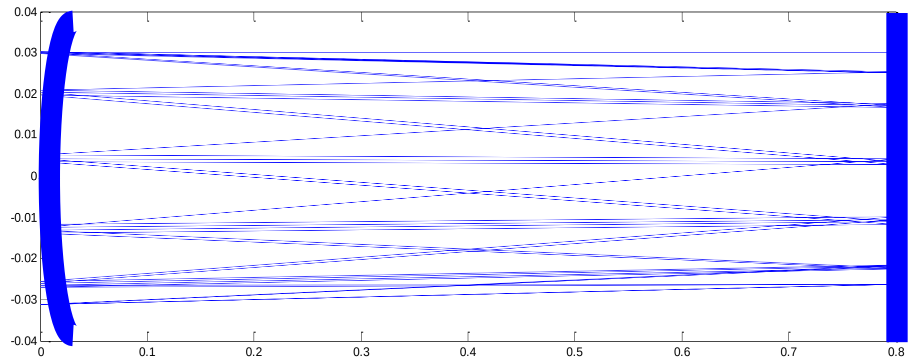
$$\phi = \tan^{-1}\left(\frac{0.5426}{0.84}\right) = 0.5735$$

$$N = \frac{2\pi}{\phi} = \frac{2\pi}{.5735} \approx 11$$

$$M^{11} \approx \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Numerical Method of Determining the Number of RTs to Image

A ray launched parallel to the optical axis will oscillate back and forth across the resonator cavity.

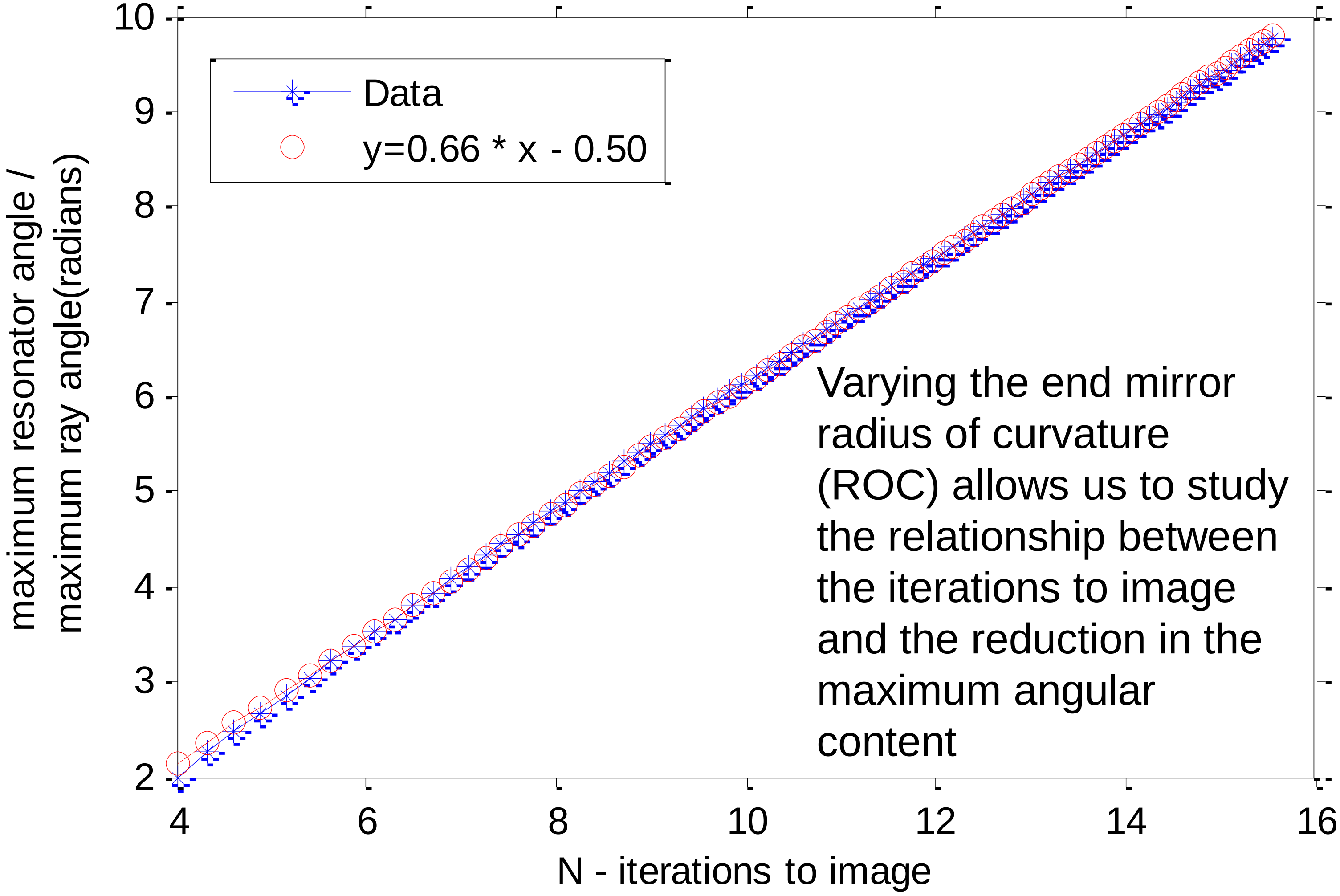


$$\text{Round-Trip ABCD Matrix} = M_{rt} = \begin{bmatrix} 0.8400 & 1.4720 \\ -0.200 & 0.8400 \end{bmatrix}$$

$$\left(M_{rt}\right)^{10.955612} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Rays self-replicate every ~11 round trips through the cavity

Angular Bandwidth Requirement Dependence on RTs to Image

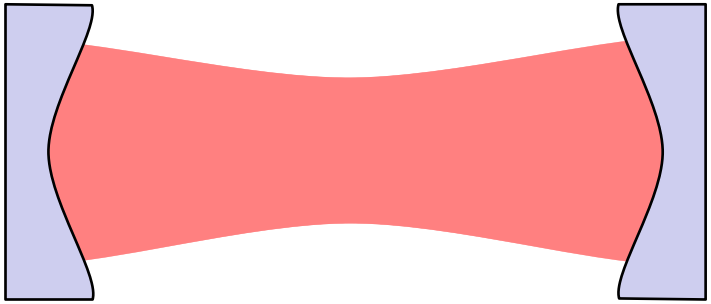


Varying the end mirror radius of curvature (ROC) allows us to study the relationship between the iterations to image and the reduction in the maximum angular content

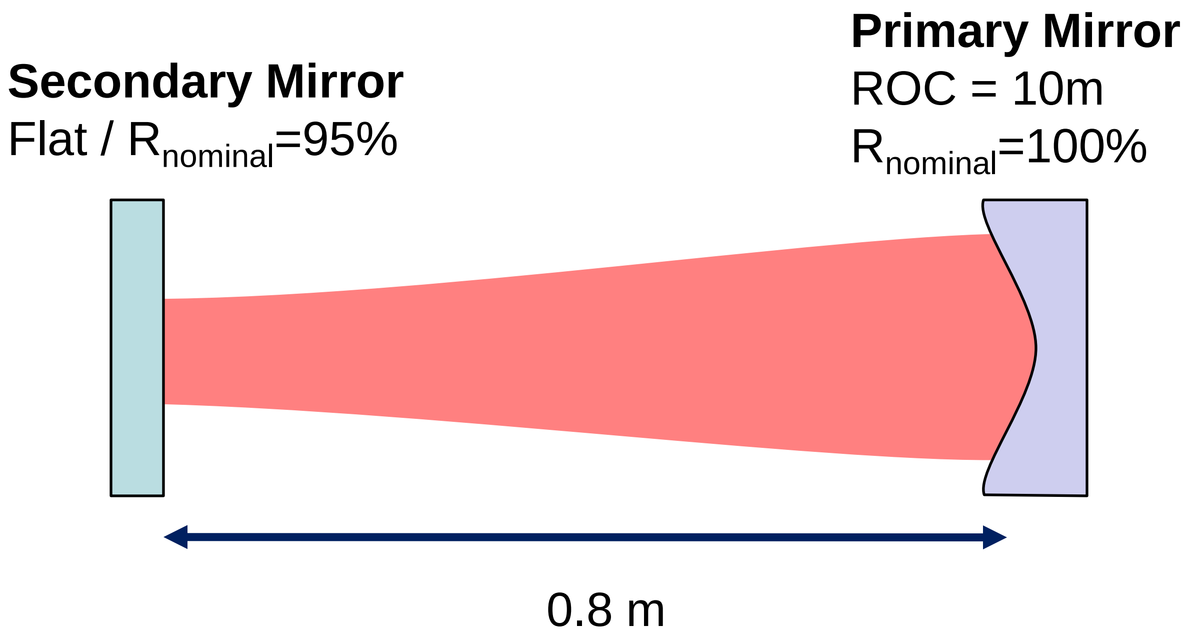
Wave-Optics Mesh Initial Conclusions

- When modeling a stable resonator, the reduction in required angular bandwidth can be reduced by approximately the number of round-trips (iterations) required to image times 2/3.
- The number of round-trips required to reimage can be determined
  - numerically,
  - graphically, or
  - analytically (through eigen analysis).

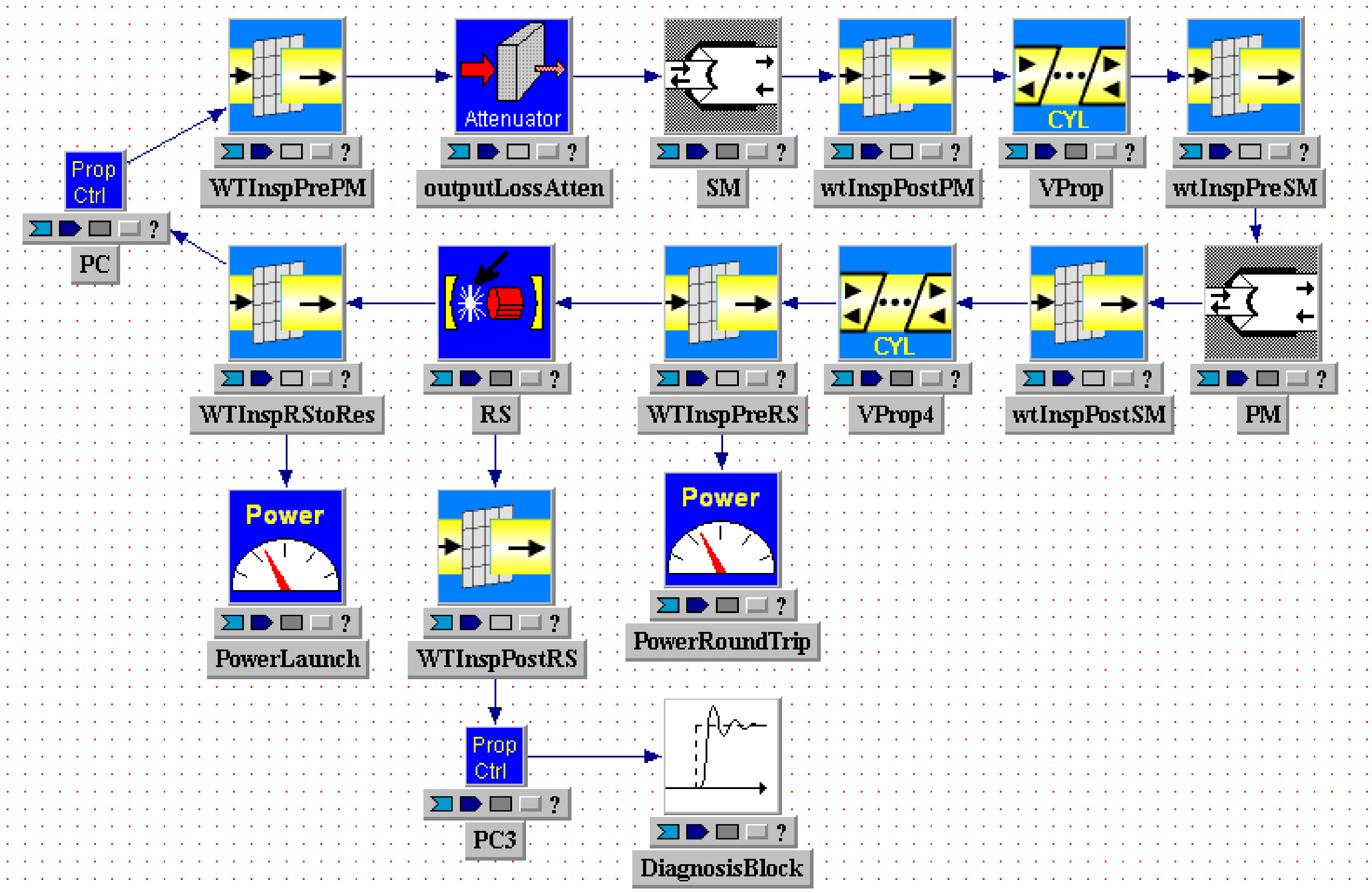
Stable Resonator Without Gain



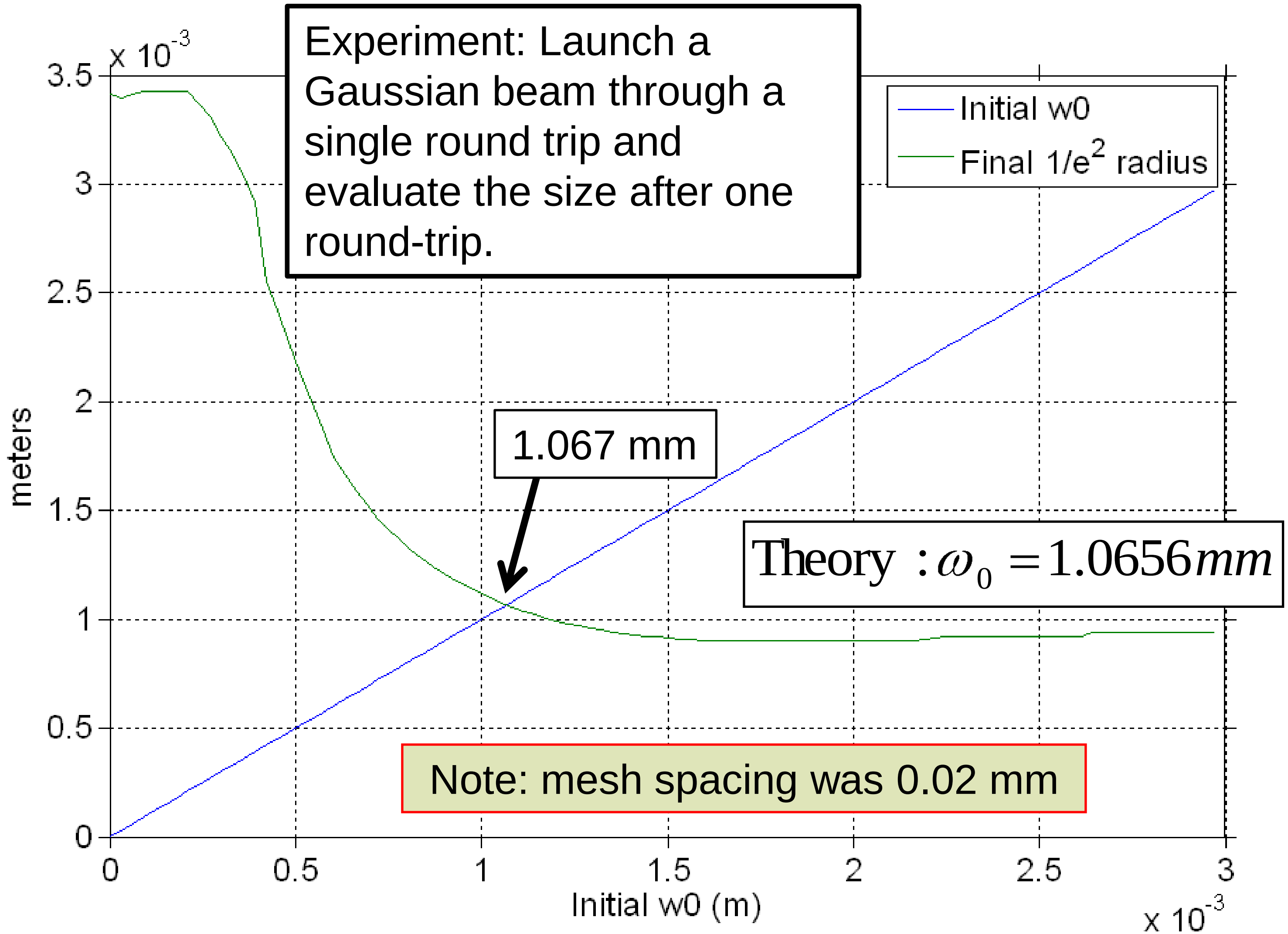
Example Laser Resonator Setup



Wave Train Model



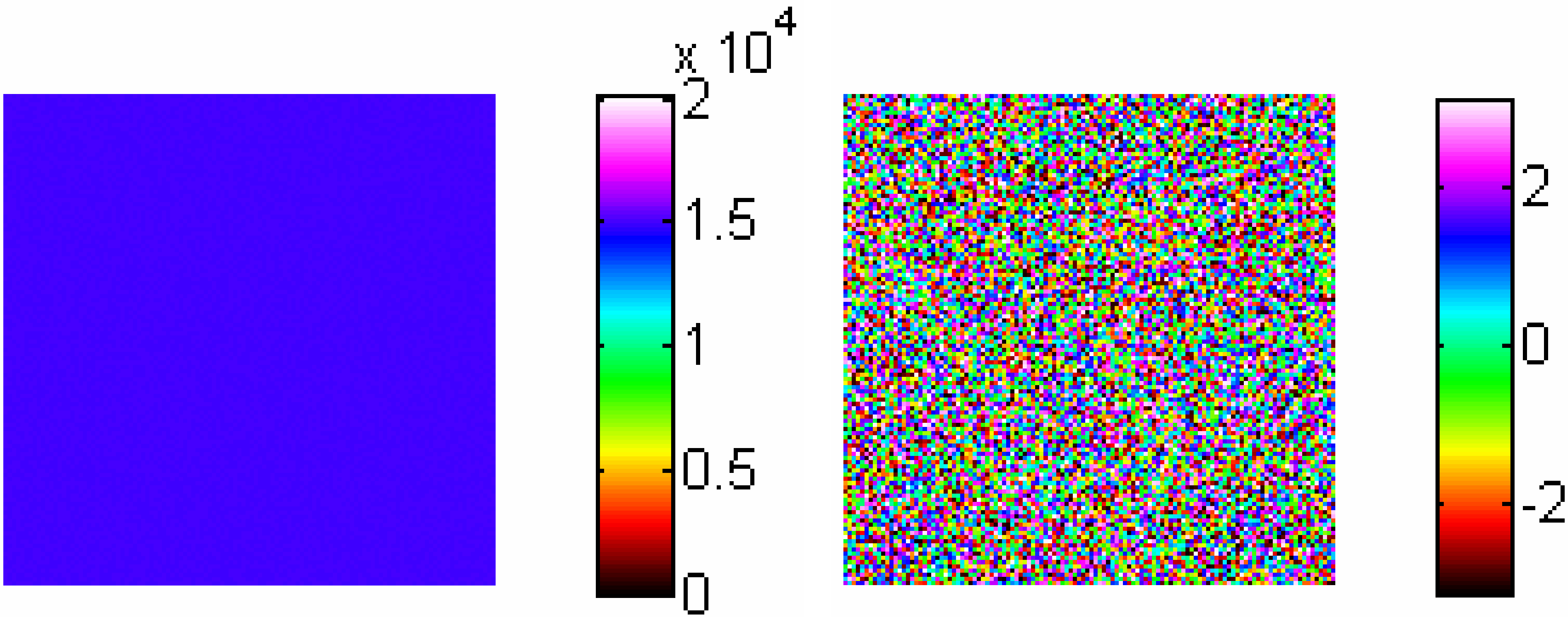
Gaussian that reproduced itself in a round-trip matched theory.



Parameters

- R_c (Radius of curvature of secondary mirror) = 10 m
- Cavity Length = 0.8 m
- Propagation grid = 256 by 56 µm (14.3 mm diameter)
- Wavelength = 1 µm
- Reflectivity of output mirror = 95 %
- Initial Field = BwomikTopHat field of 6 cm initial Radius and 15000 amplitude
- Normalization = 1
- Iterations = 10000
- Varying Aperture diameter

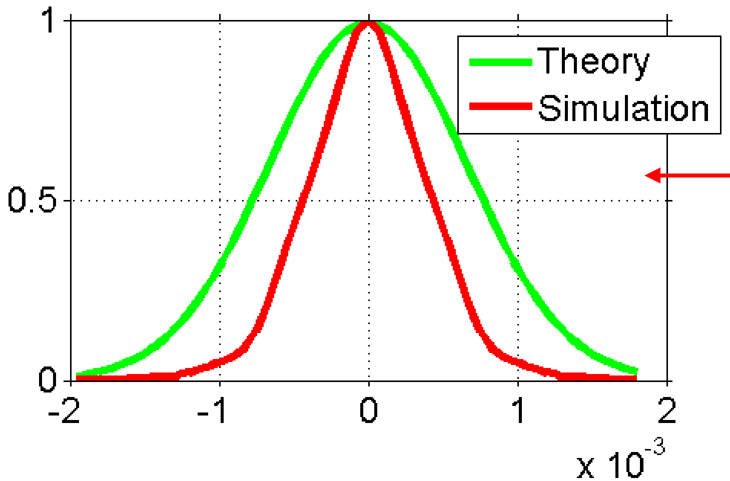
Seeded all laser modes initially with a Bwomik field.

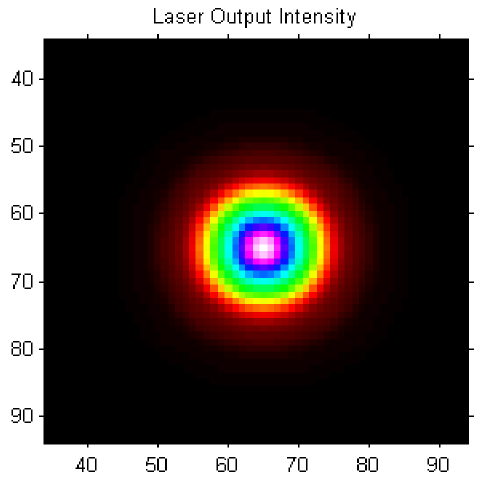


Bwomik Field is a plane wave with random phase that tends to seed all the modes of a resonator. This is implemented in “LaserGridInitializers.h”.

Converged Resonator Field Dependence on Internal Aperture Diameter

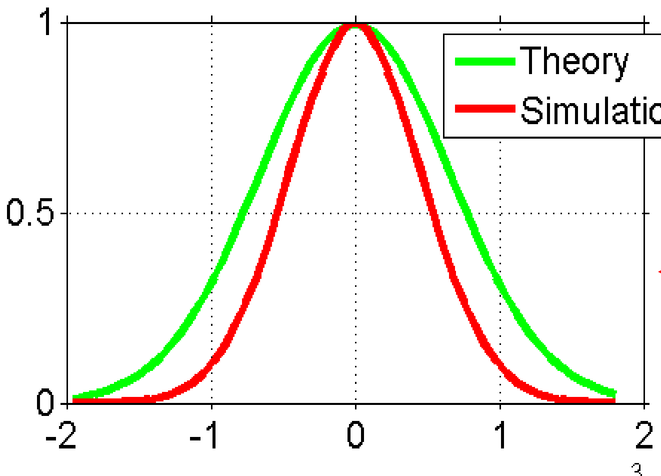
Larger aperture results approximate the theory more accurately.

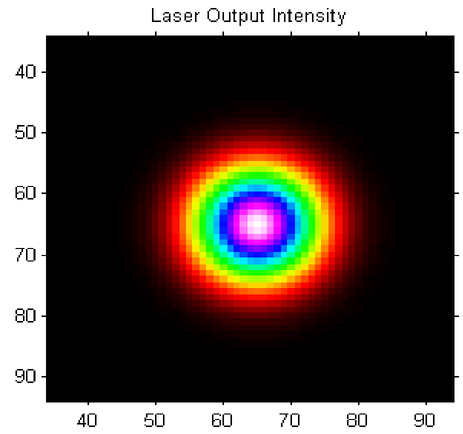




Iterations = 10000

Aperture diameter = 2.0 mm





Aperture diameter = 4.0 mm

NOTE: Theory is TEM$_{00}$ shape.

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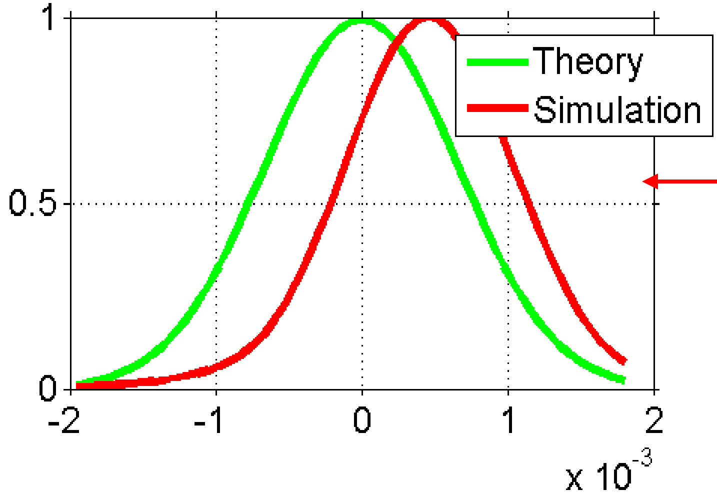
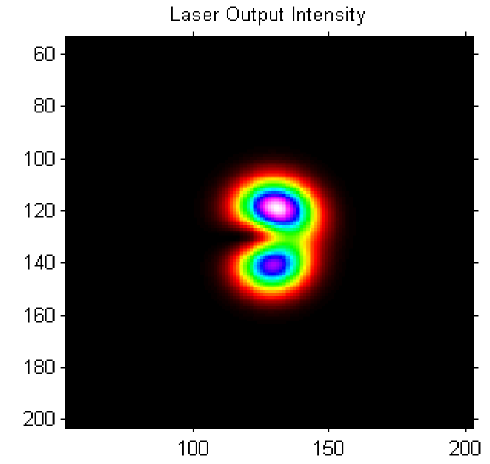
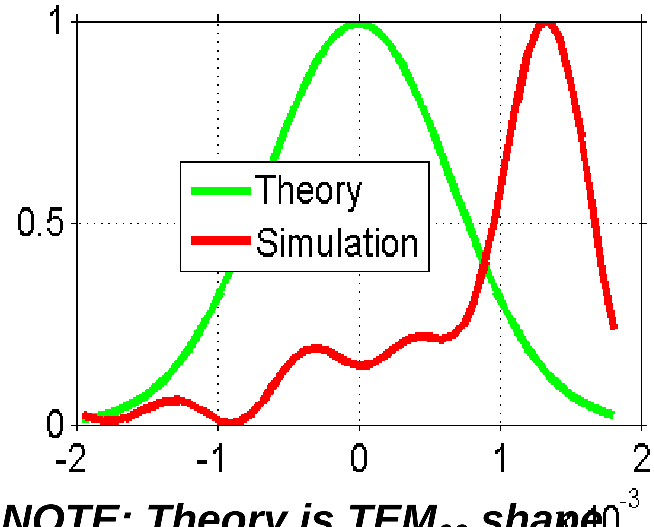
Bigger apertures require more iterations to converge.

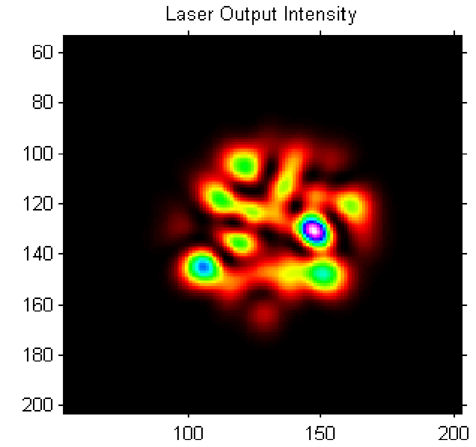




Iterations = 10000

Aperture diameter = 5.0 mm

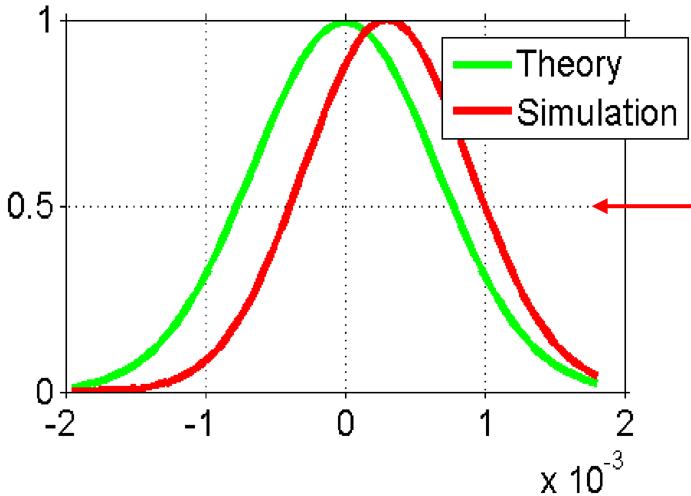


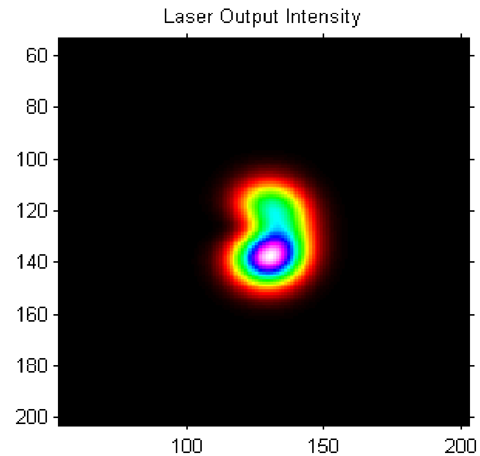


Aperture diameter = 7.0 mm

NOTE: Theory is TEM$_{00}$ shape.

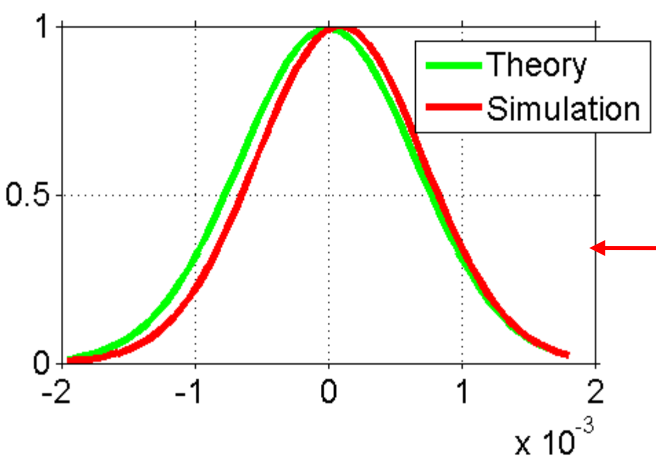
The 5-mm case begins to approach convergence after 50,000 iterations.

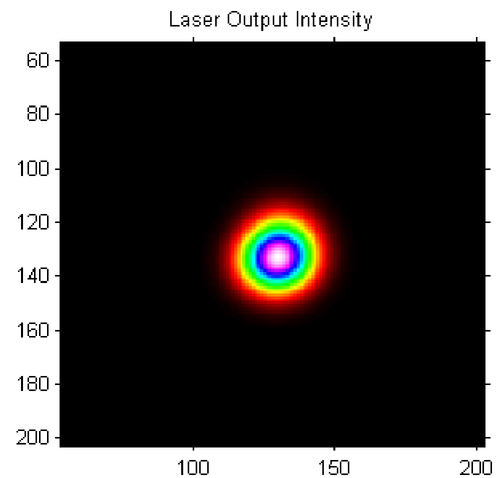




Iterations = 25000

Aperture diameter = 5.0 mm





Iterations = 50000

Aperture diameter = 5.0 mm

NOTE: Theory is TEM_{00} shape.

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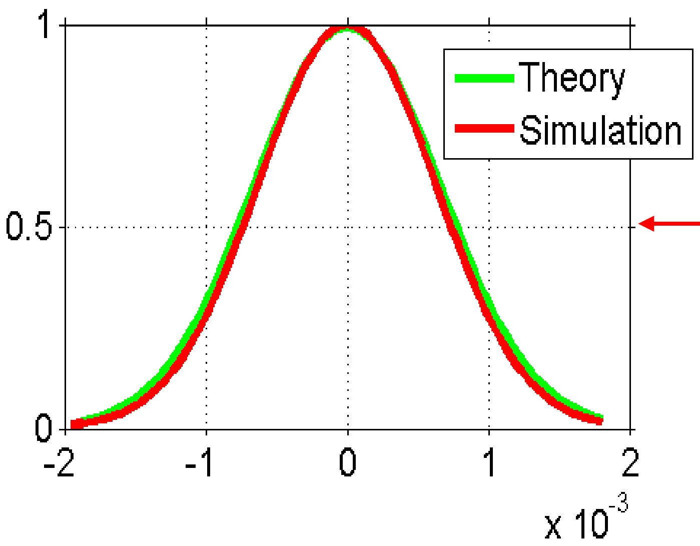
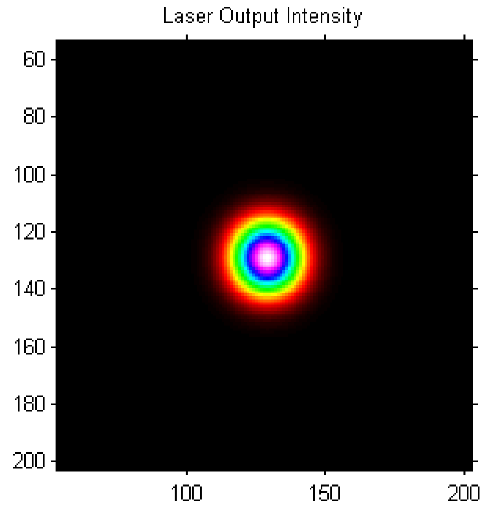
The 5mm diameter aperture was reasonably well converged after 75,000 iterations.





Iterations = 75000

Aperture diameter = 5.0 mm

75,000 iterations takes about 13 hours

NOTE: Theory is TEM_{00} shape.

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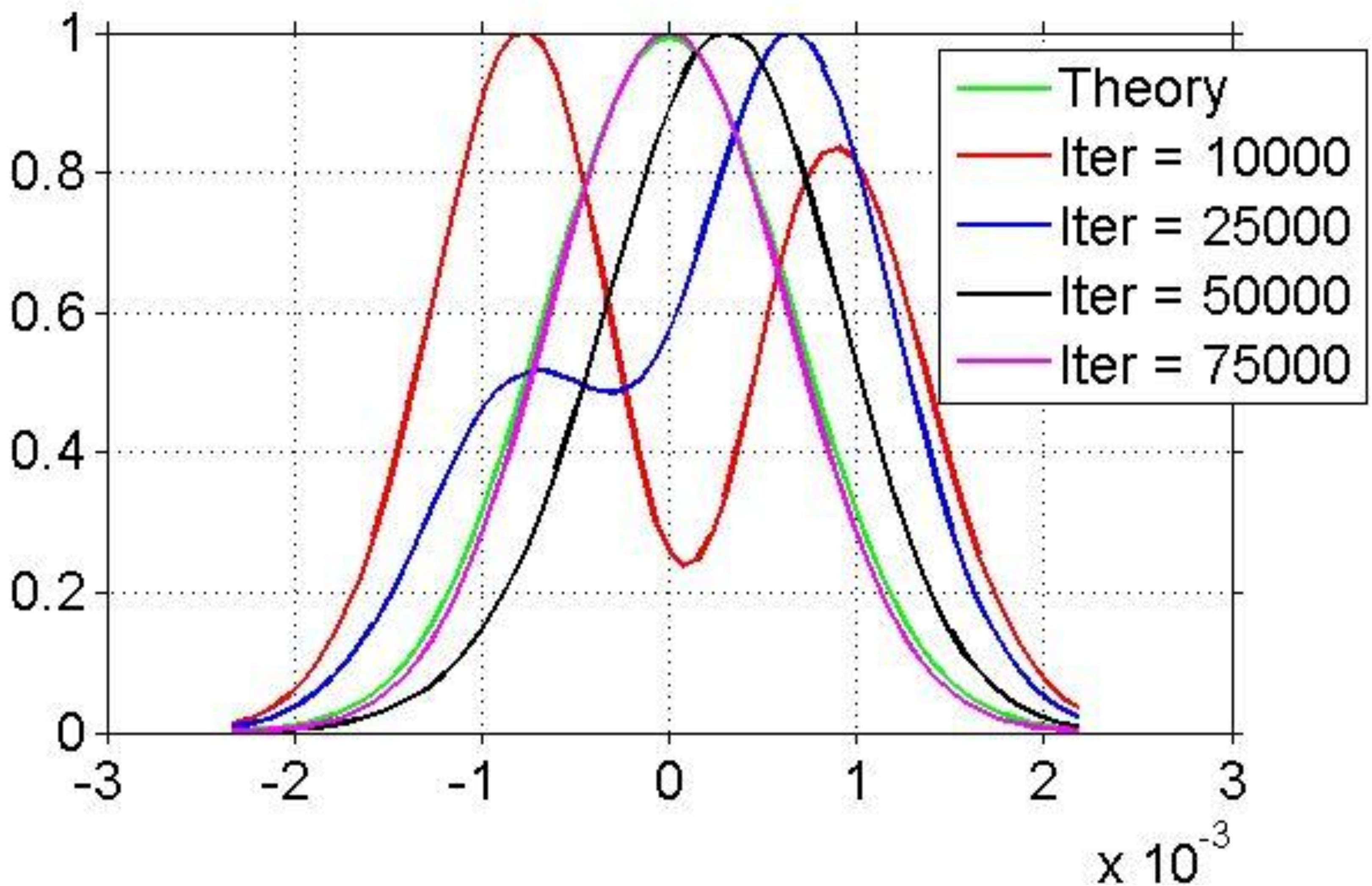
Intensity Cross-Section Comparison for 5-mm Aperture Cases with Different Iteration Numbers



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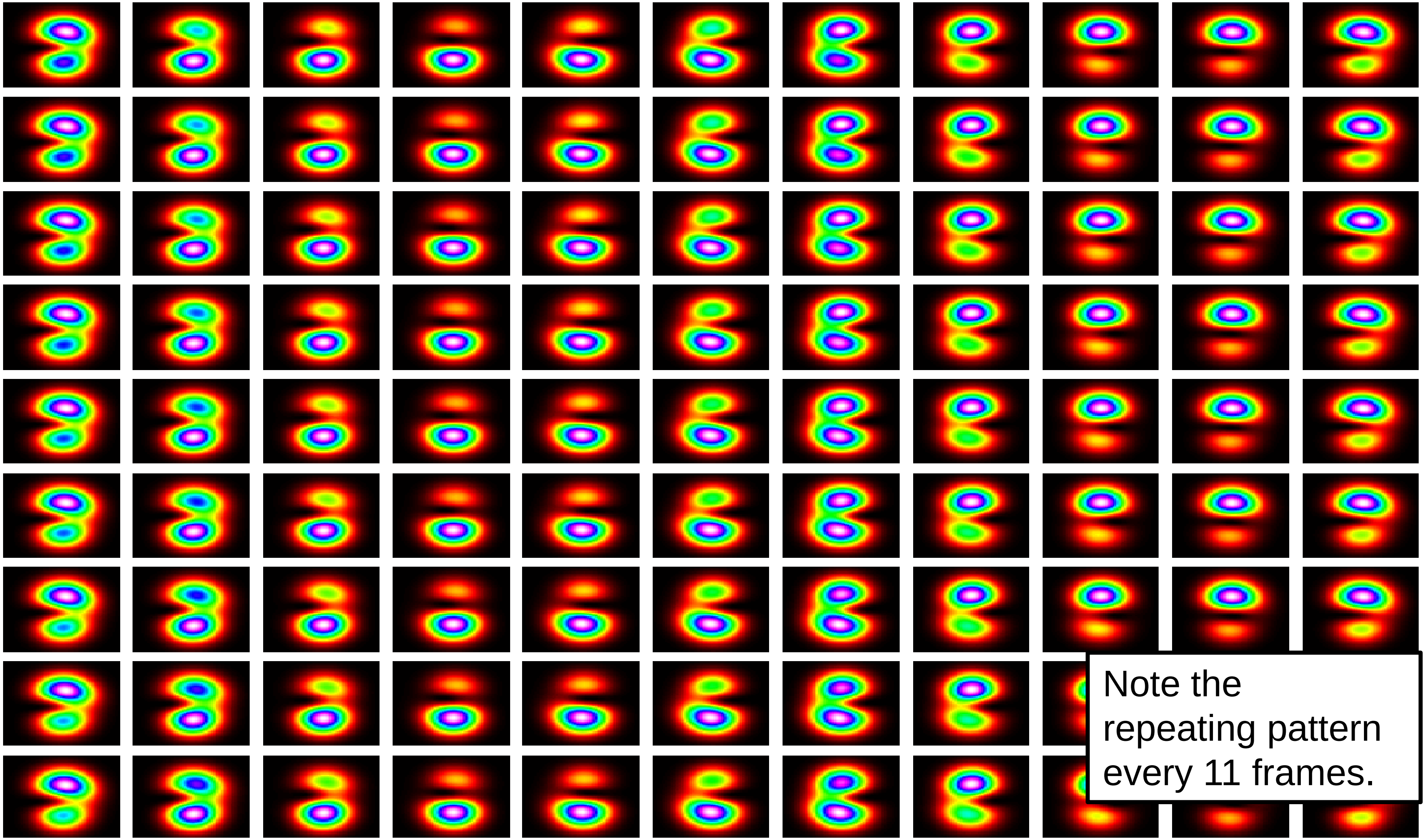
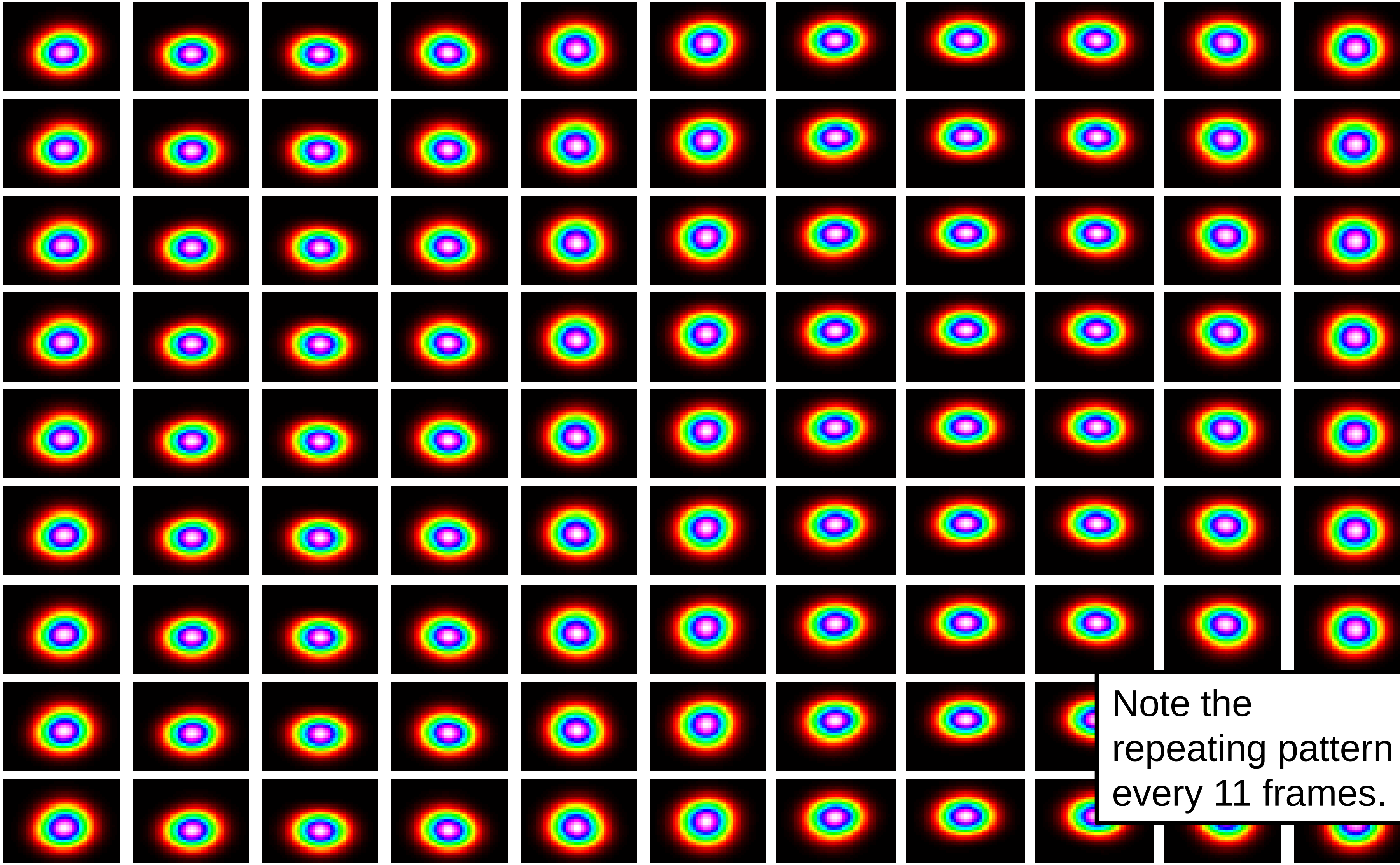
Variation of Output Intensity with Last 99 Frames of 10,000 with 5-mm Diameter Aperture



Note the repeating pattern every 11 frames.

Variation of Output Intensity with Last 99 Frames of 50,000 with 5-mm Diameter Aperture



Note the repeating pattern every 11 frames.

jmansell@mza.com

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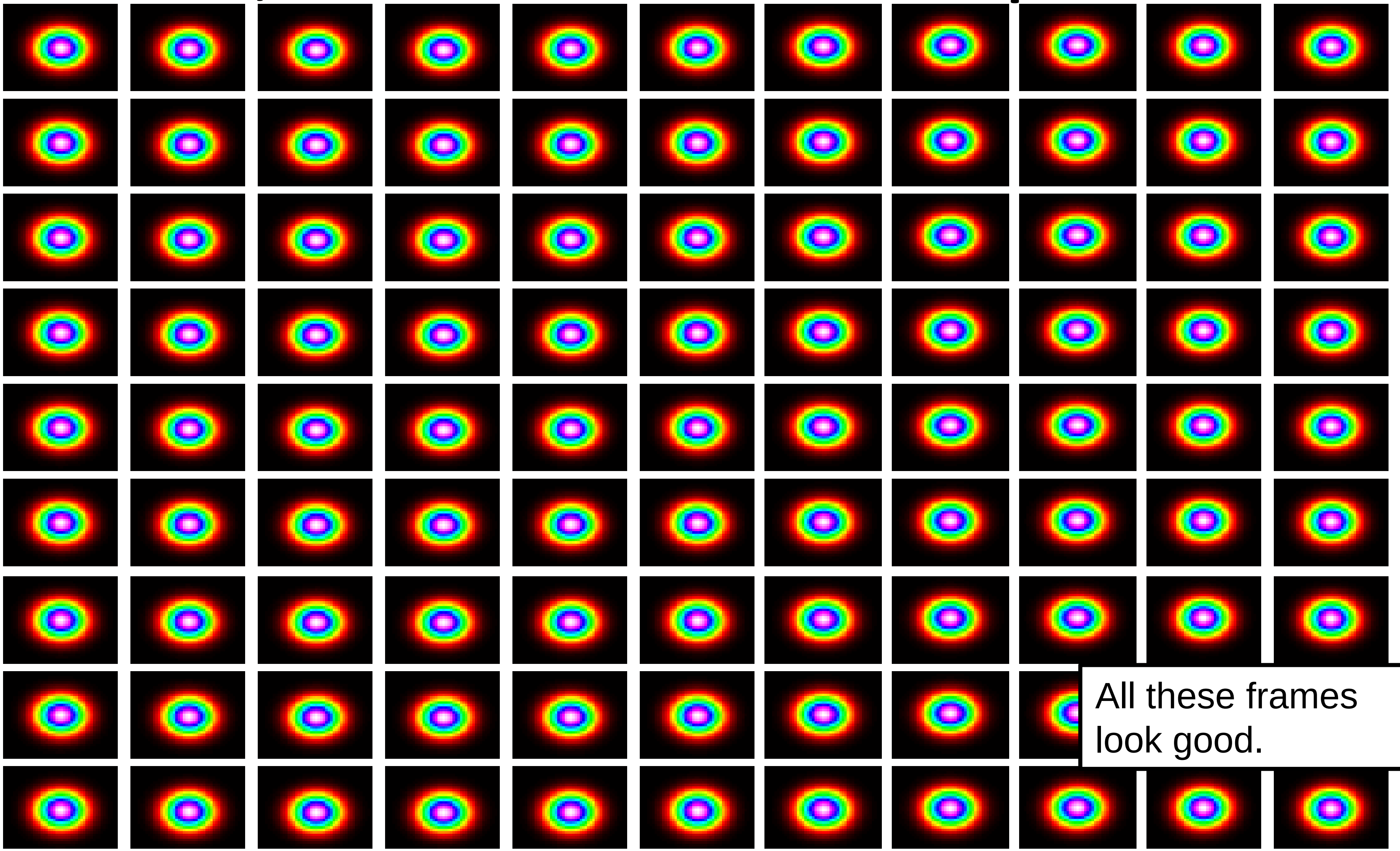
Variation of Output Intensity with Last 99 Frames of 75,000 with 5-mm Diameter Aperture



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Stable Resonator Model Conclusions

- The WaveTrain stable resonator model showed that the models with larger aperture (~5 times w_0) converged very close to the theoretical shape in many (75,000) iterations.
- Even fairly early in the iterations, the beam intensity shape repeats itself every 11 iterations, as is predicted by theory.

Conclusions

- In stable resonator modeling, the number of round-trip iterations to image directly impacts the wave-optics mesh requirements.
- We showed three different techniques for determining the number of round-trips to image: analytical, graphical, and numerical.
- Using this theory, we have shown that our stable resonator model without gain matches well with the theoretical predictions.

Future Work and Acknowledgements

- We need to perform more anchoring to experimental data to complete the verification of this new technique.
  - Experiment: Aperture in a small stable resonator.
- We need to consider how the different frequencies impact model performance.
- Acknowledgements
  - A. Paxton and A. E. Siegman for technical discussions
  - Funded via the LADERA contract

Questions?

jmansell@mza.com

(505) 245-9970 x122