Integrating Wave-Optics and 5x5 Ray Matrices for More Accurate Optical System Modeling

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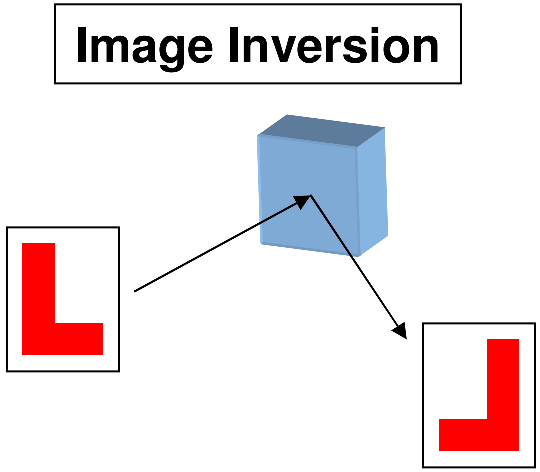
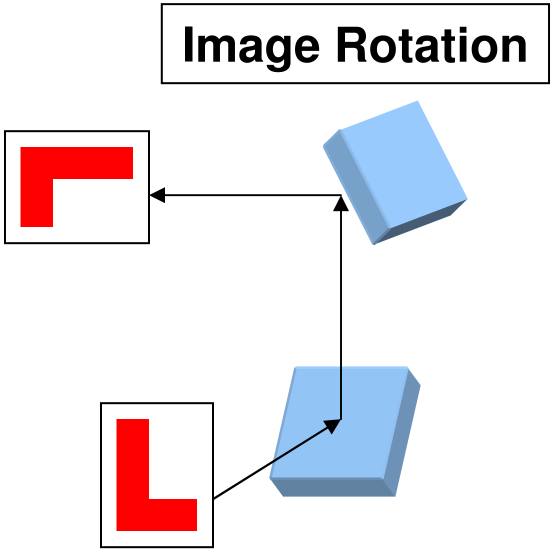
Outline

- Motivation
- Methodology
  - Simplified Ray Tracing
  - 5x5 Ray Matrix Formalism
- Integration of 5x5 Ray Matrices with Wave-Optics
- Conclusions & Future Work

Motivation 1: Model More Effects

- Wave-optics is not sufficient to model some optical effects like
  - reflection-induced image inversion and
  - geometric image rotation.





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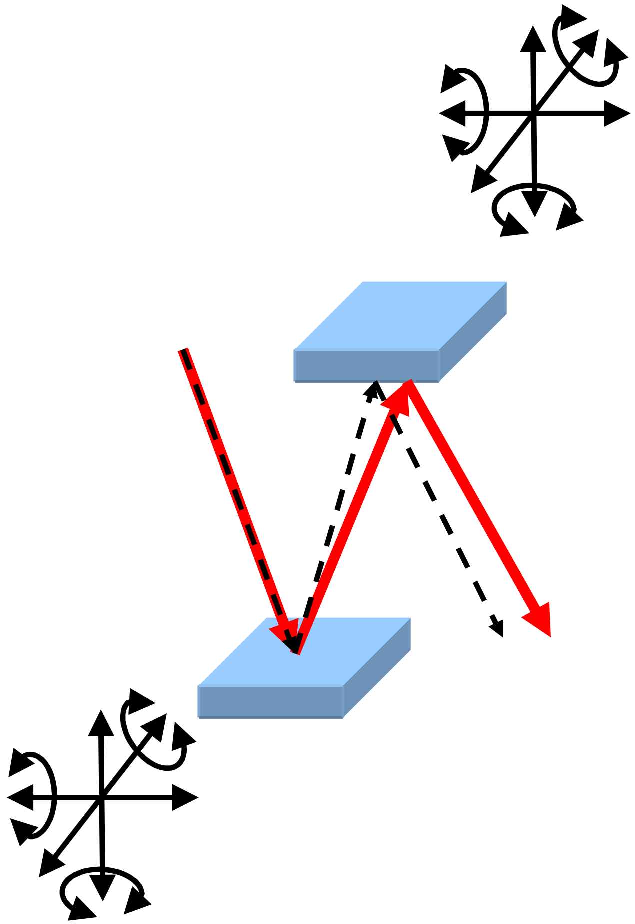
Motivation 2: 6 DOF Perturbation Analysis

- Mechanical disturbances of optics and the resulting induced beam jitter control are typically modeled separately from wave-optics.
- These effects impact wave-optics performance.
- We want a mechanism for control model integration.



Our Solution

- Augment our wave-optics with a 5x5 ray matrix formalism based on simplified ray tracing.
- Why?
  - Wave-optics infrastructure exists
  - Complete ray tracing is complex and time consuming
  - 5x5 ray matrices
    - simple to implement,
    - fast,
    - sufficient to add desired effects
  - Consistent with
    - beam jitter control matrix development and
    - FEM modeling

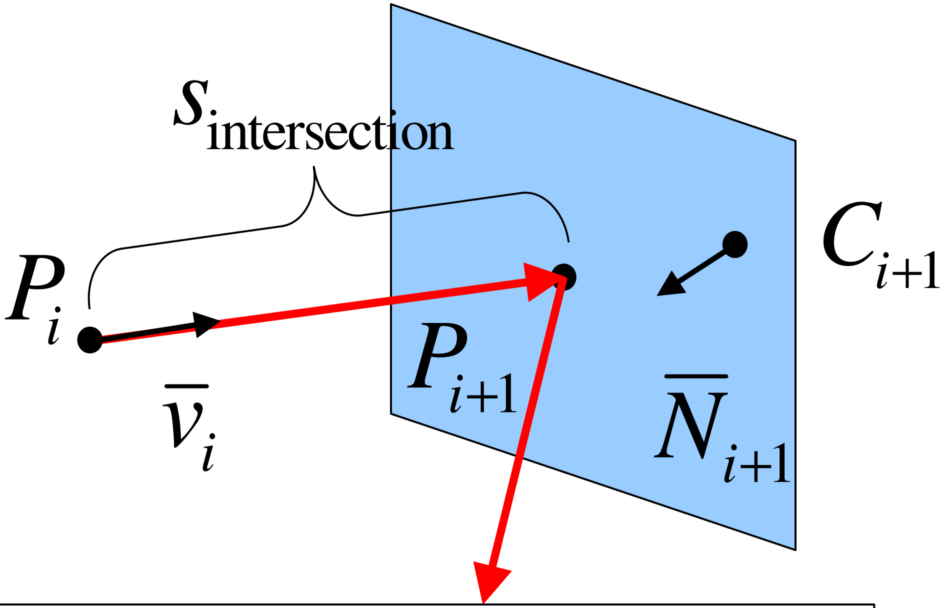
Presentation Limitations

- Removed some mathematical methods that are excessively time-consuming to present.
  - More time to focus on highlights
- These details are available in the paper.

Simplified Ray Tracing

Ray Trace Procedure

- Start with a coordinate in a plane.
- Find the intersection of that coordinate in the next plane.
- Find next direction through interaction with plane normal plus curvature.



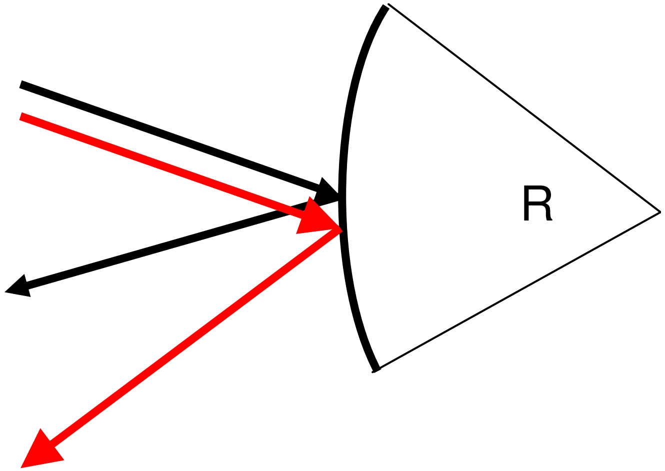
$$line = \{P_i, \bar{v}_i\},\ plane = \{C_{i+1}, \bar{N}_{i+1}\}$$

$$s_{\text{intersection}} = \left(-\frac{\bar{N}_{i+1} \bullet (P_i - C_{i+1})}{\bar{N}_{i+1} \bullet \bar{v}_i} \right)$$

$$P_{\text{i+1}} = P_i + \left(s_{\text{intersection}}\right) \cdot \bar{v}_i$$

Curvature-Induced Tilt

- Additional tilt added due to optic curvature is handled separately by
  – calculating the distance from the center of the optic (called beam walk) and
  – adding the appropriate curvature-induced tilt.



$$f = R/2$$

$$\Delta\theta_{local} = \begin{bmatrix} 0 & 1/f \\ -1/f & 0 \end{bmatrix} \cdot bw_{i+1}^{\ \ local}$$

$$\Delta\theta_{global} = M_{transform} \cdot \Delta\theta_{local}$$

$$\overline{v}_{i+1} = \overline{v}_{i+1} + \Delta\theta_{global} \times \overline{v}_{i+1}$$

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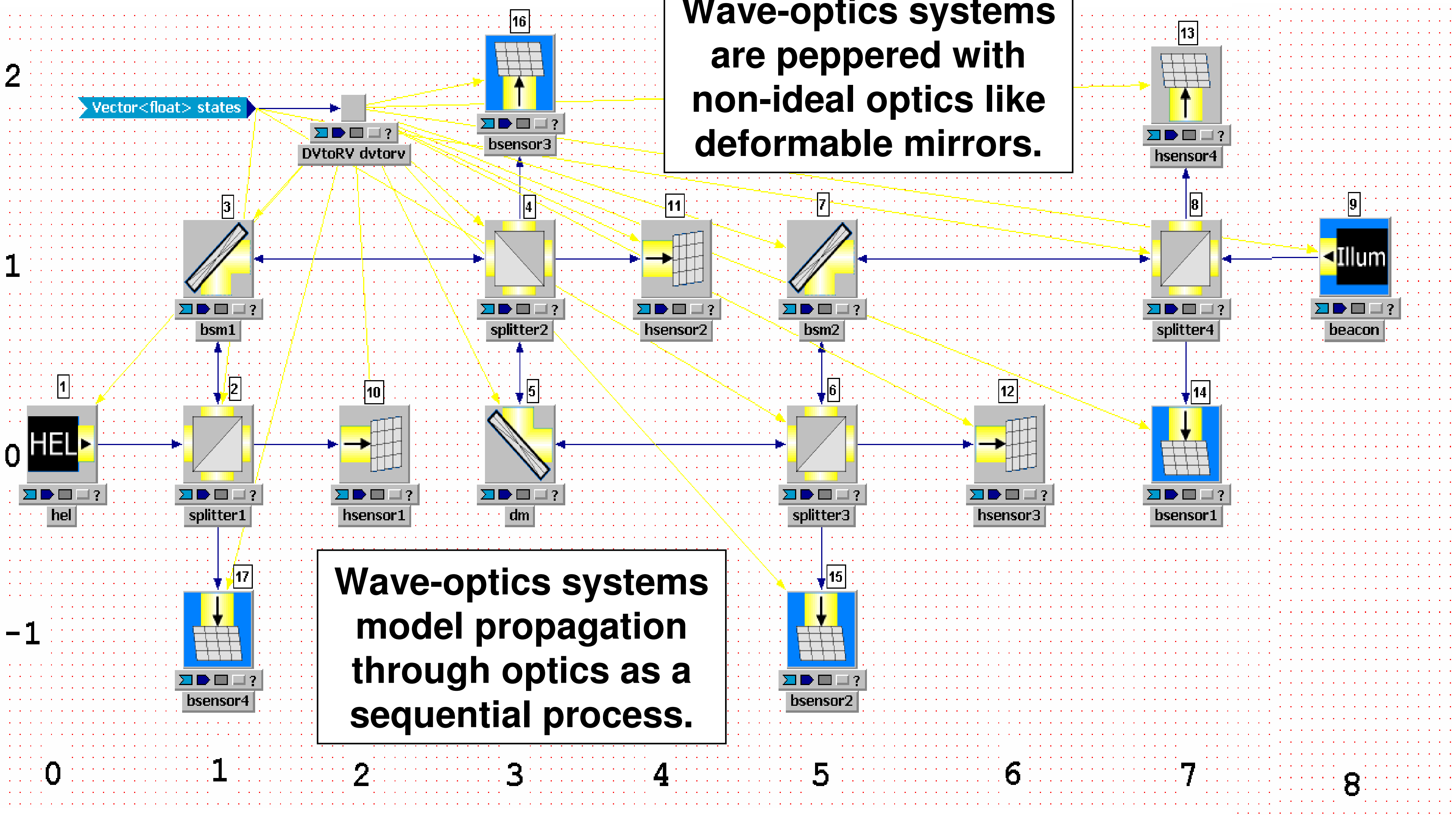
Sequential Evaluation Motivation



Wave-optics systems are peppered with non-ideal optics like deformable mirrors.

Wave-optics systems model propagation through optics as a sequential process.

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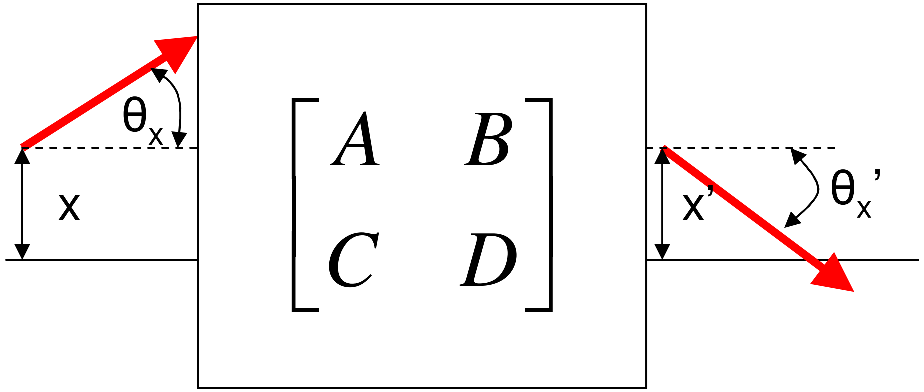
Generalizing Ray Tracing

- We were looking for a way to:
  – apply ray tracing to a general ray and
  – apply it sequentially to an optical system
- Needed Effects:
  – image rotation
  – image inversion
  – ability to do perturbation analysis

Ray Matrix Formalism

Introduction - ABCD Matrices

- The most common ray matrix formalism is the 2x2 or ABCD that describes how a ray height, x, and angle, θ_x, changes through a system.



$$\begin{bmatrix} x' \\ \theta_x' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ \theta_x \end{bmatrix}$$

$$x' = Ax + B\theta_x$$

$$\theta_x' = Cx + D\theta_x$$

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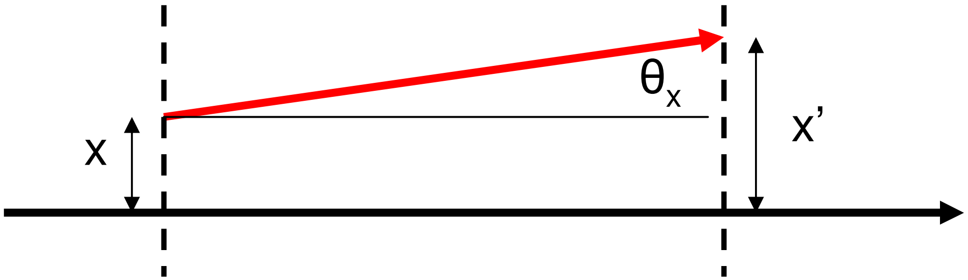
2x2 Ray Matrix Examples

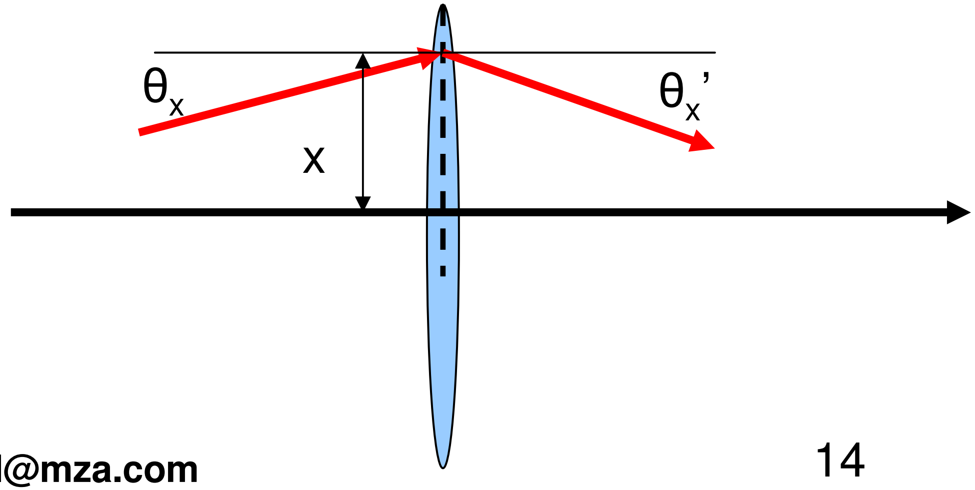
Propagation



$$x' = x + \theta_x \bullet L$$

$$\begin{bmatrix} x' \\ \theta_x' \end{bmatrix} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \theta_x \end{bmatrix}$$

Lens



$$\theta_x' = \theta_x - x / f$$

$$\begin{bmatrix} x' \\ \theta_x' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} x \\ \theta_x \end{bmatrix}$$

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Example ABCD Matrices

Matrix Type	Form	Variables
Propagation	$\begin{bmatrix} 1 & L/n \\ 0 & 1 \end{bmatrix}$	L = physical length n = refractive index
Lens	$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$	f = effective focal length
Curved Mirror (normal incidence)	$\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}$	R = effective radius of curvature
Curved Dielectric Interface (normal incidence)	$\begin{bmatrix} 1 & 0 \\ -(n_2 - n_1)/R & 1 \end{bmatrix}$	n_1 = starting refractive index n_2 = ending refractive index R = effective radius of curvature

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3x3 and 4x4 Formalisms

- Siegman's *Lasers* book describes two other formalisms: 3x3 and 4x4
- The 3x3 formalism added the capability for tilt addition and off-axis elements.
- The 4x4 formalism included two-axis operations like axis inversion and image rotation.

3 x 3

$$\begin{bmatrix} A & B & E \\ C & D & F \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \theta_x \\ 1 \end{bmatrix}$$

E = Offset

F = Added Tilt

4 x 4

$$\begin{bmatrix} A_x & B_x & & \\ C_x & D_x & & \\ & & A_y & B_y \\ & & C_y & D_y \end{bmatrix} \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \end{bmatrix}$$

5x5 Formalism

- We developed a 5x5 ray matrix formalism as a combination of the 2x2, 3x3, and 4x4.

$$\begin{bmatrix} A_x & B_x & 0 & 0 & E_x \\ C_x & D_x & 0 & 0 & F_x \\ 0 & 0 & A_y & B_y & E_y \\ 0 & 0 & C_y & D_y & F_y \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \\ 1 \end{bmatrix}$$

Example 5x5 System Matrices

Matrix Type	Form	Variables
Image Rotation	$\begin{bmatrix} C & 0 & S & 0 & 0 \\ 0 & C & 0 & S & 0 \\ -S & 0 & C & 0 & 0 \\ 0 & -S & 0 & C & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	C = cos(θ_{image}) S = sin(θ_{image}) θ_{image} = image rotation angle
Magnification	$\begin{bmatrix} M_x & 0 & 0 & 0 & 0 \\ 0 & 1/M_x & 0 & 0 & 0 \\ 0 & 0 & M_y & 0 & 0 \\ 0 & 0 & 0 & 1/M_y & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	M_x = magnification along x axis M_y = magnification along y axis
Reflection Inversion (reflection from a mirror in the plane of the x-axis)	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	

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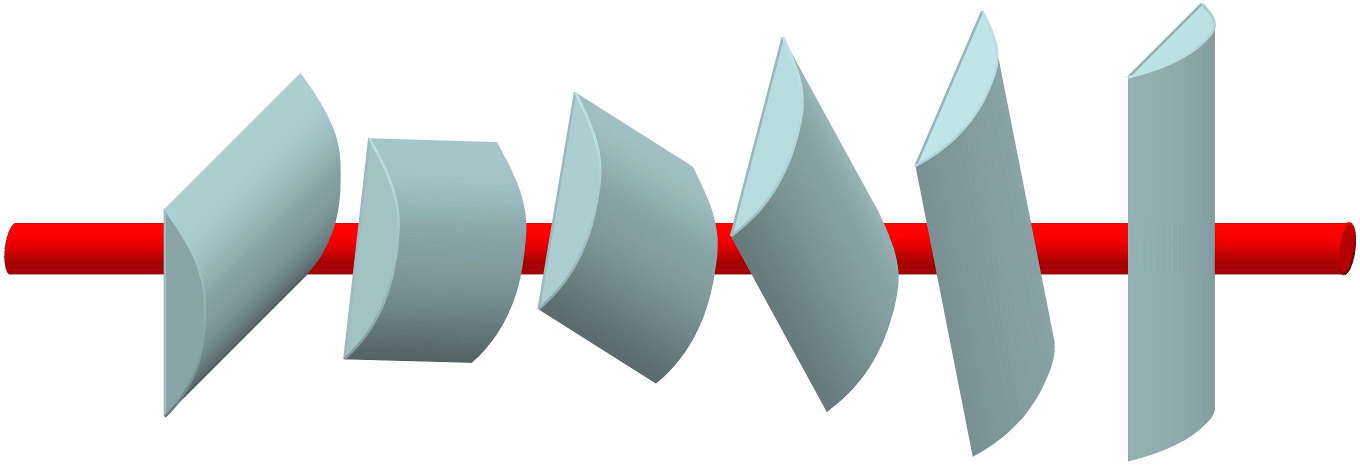
Example 5x5 System Matrices 2

Matrix Type	Form	Variables
Image Translation	$$\begin{bmatrix} 1 & 0 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \Delta y \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$	Δx = translation along the x axis Δy = translation along the y axis
Tilt Addition	$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta\theta_x \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta\theta_y \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$	$\Delta\theta_x$ = added tilt along the x axis $\Delta\theta_y$ = added tilt along the y axis

Limitations

- No Astigmatism
  - axially symmetric curvature only
- Why?
  - This can be represented, but there may be insufficient degrees of freedom for arbitrary axis astigmatism.
  - Astigmatism can be put into the wave-optics.



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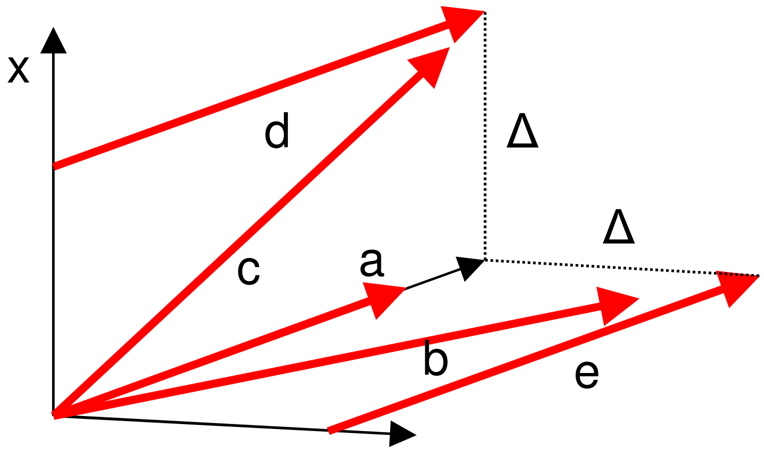
Procedure for Generation of a 5x5 Ray Matrix

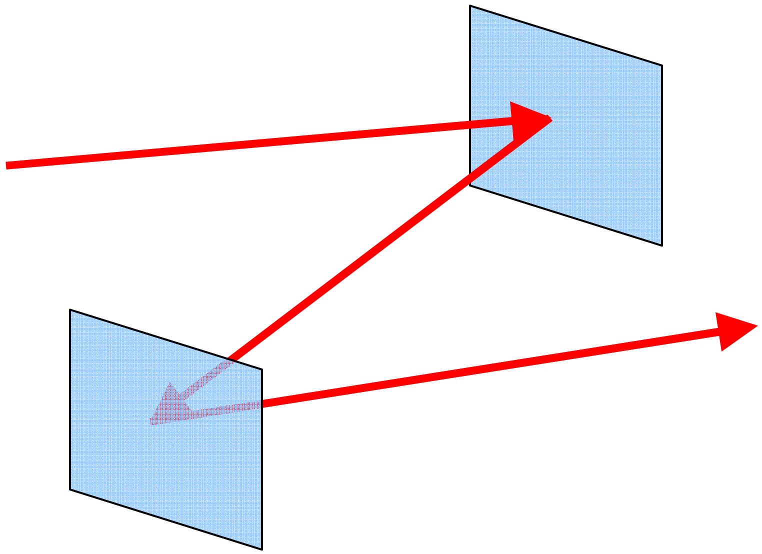
Motivation for Modified Procedure

- Multiplying 5x5 ray matrices allow for all the desired effects to be accumulated
- The magnitude of the image rotation cannot be determined with a sequential ray matrix approach
- A different procedure had to be devised to incorporate this effect

Simple Ray Trace Procedure

- 5 probe rays
- Rays modeled as simple geometric ray tracing
  - thin optic approximation
  - curvature handled separately





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Reduction of Probe Rays to 5x5 Matrix

- Determine the difference in location and direction from the unperturbed central ray (a).
- Project these differences onto the x and y beam axes in global coordinates.
- RESULT: 5x5 matrix

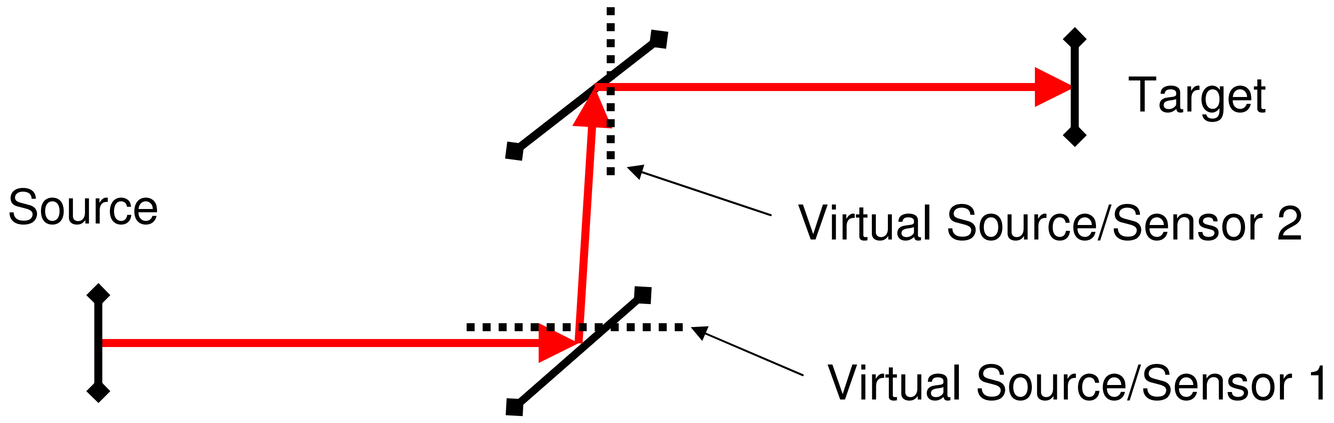
$$\Delta v_b = \bar{v}_{last}(b) - \bar{v}_{last}(a) \qquad \Delta P_b = P_{last}(b) - P_{last}(a)$$
$$\Delta v_c = \bar{v}_{last}(c) - \bar{v}_{last}(a) \qquad \Delta P_c = P_{last}(c) - P_{last}(a)$$
$$\Delta v_d = \bar{v}_{last}(d) - \bar{v}_{last}(a) \qquad \Delta P_d = P_{last}(d) - P_{last}(a)$$
$$\Delta v_e = \bar{v}_{last}(e) - \bar{v}_{last}(a) \qquad \Delta P_e = P_{last}(e) - P_{last}(a)$$

$$\begin{bmatrix} A1 & A2 & A3 & A4 & A5 \\ B1 & B2 & B3 & B4 & B5 \\ C1 & C2 & C3 & C4 & C5 \\ D1 & D2 & D3 & D4 & D5 \\ E1 & E2 & E3 & E4 & E5 \end{bmatrix}$$

$$A1 = \Delta P_d \bullet \bar{x}_{beam}^{global} \qquad B1 = \Delta v_d \bullet \bar{x}_{beam}^{global}$$
$$A2 = \Delta P_b \bullet \bar{x}_{beam}^{global} \qquad B2 = \Delta v_b \bullet \bar{x}_{beam}^{global}$$
$$A3 = \Delta P_e \bullet \bar{x}_{beam}^{global} \qquad B3 = \Delta v_e \bullet \bar{x}_{beam}^{global} \qquad \ldots$$
$$A4 = \Delta P_c \bullet \bar{x}_{beam}^{global} \qquad B4 = \Delta v_c \bullet \bar{x}_{beam}^{global}$$
$$A5 = \Delta P_a \bullet \bar{x}_{beam}^{global} \qquad B5 = \Delta v_a \bullet \bar{x}_{beam}^{global}$$

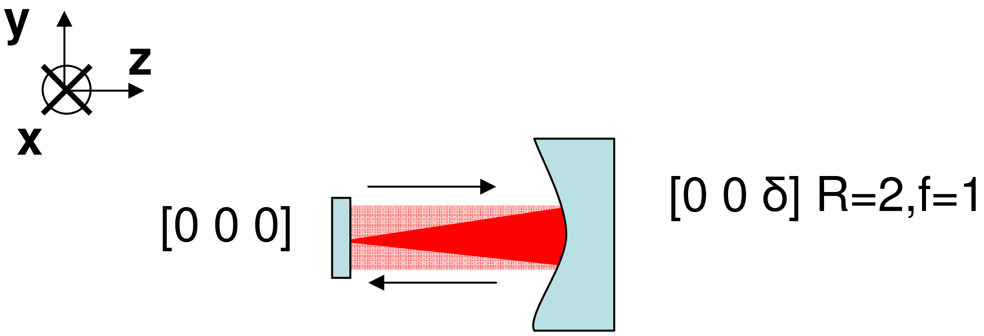
Sequential Evaluation of 5x5 Matrices

- Ray matrices can be multiplied sequentially to establish a system ray matrix.
- We established a sequential technique that involves:
  - Propagate from the source to a plane a small delta from the optic, which is a "virtual sensor".
  - That plane becomes the next "virtual source".



Example Systems Analysis using 5x5 Ray Matrices

Simple Powered Optic



Unperturbed Matrix

$$
\begin{vmatrix}
1 & 2\delta & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 2\delta & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{vmatrix}
$$

NOTE: δ terms will be removed henceforth.

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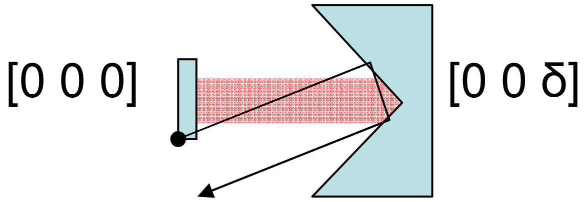
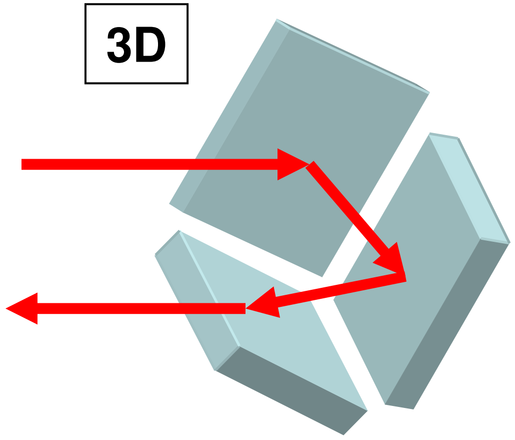
Retro-Reflector Case

Cross-Section



Only 2 of 3 bounces shown.

3D



Unperturbed Matrix

1	0	0	0	0
0	-1	0	0	0
0	0	-1	0	0
0	0	0	1	0
0	0	0	0	1

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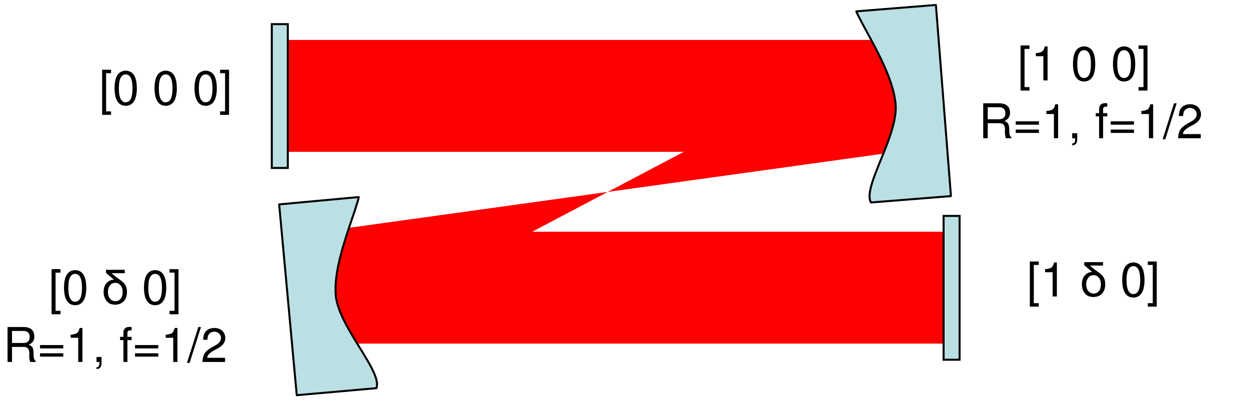
Focal Plane Inversion Plane Test Case



$$\left|\begin{matrix} -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}\right|$$

$$\left|\begin{matrix} 1 & 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}\right|$$

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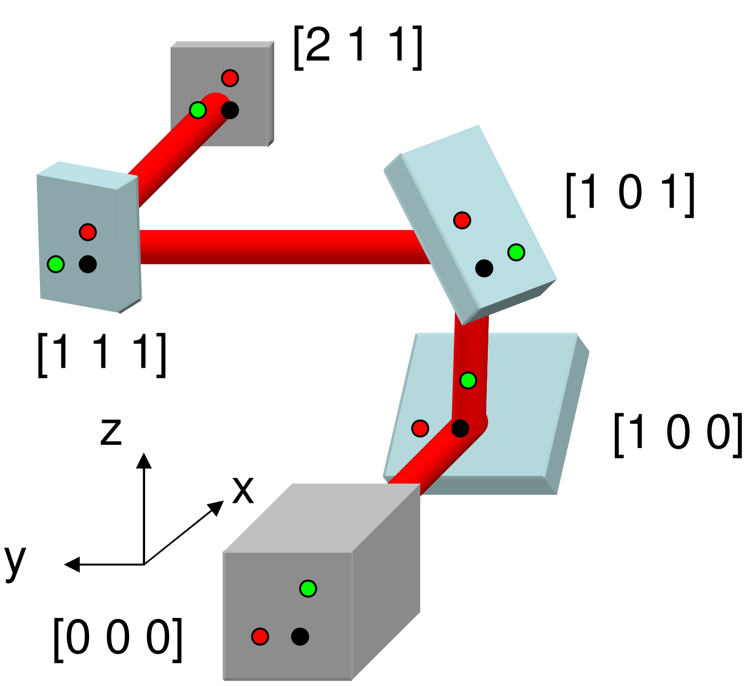
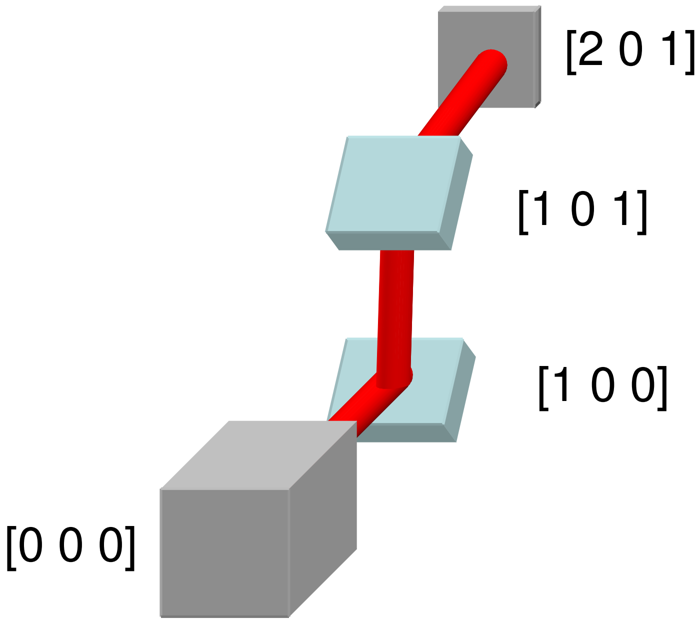
Set of 3 Image Rotation Matrices 1/2





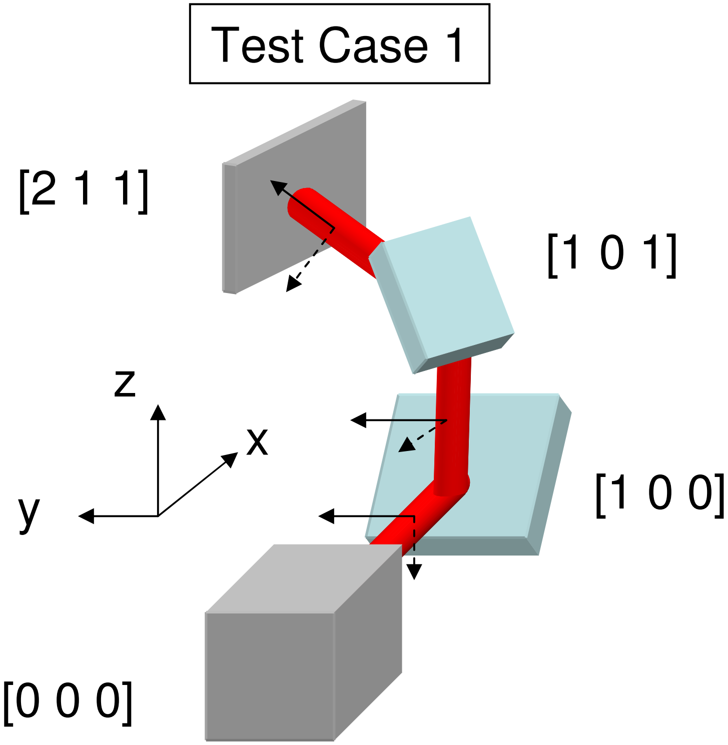
Unperturbed Matrix

$$\begin{vmatrix} 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Unperturbed Matrix

$$\begin{vmatrix} 1 & 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

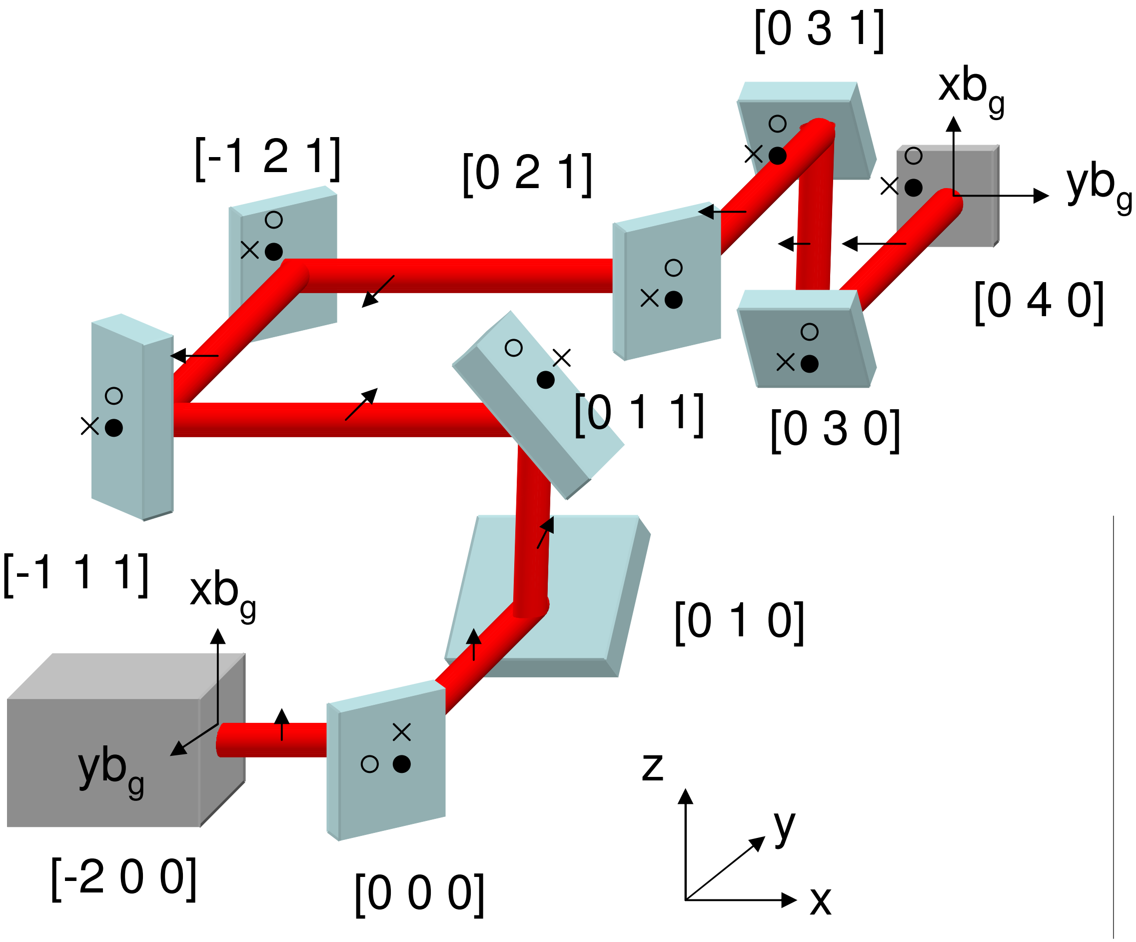
45° Image Rotation



Unperturbed Matrix

$1/\sqrt{2}$	3.12	$1/\sqrt{2}$	3.12	0
0	$1/\sqrt{2}$	0	$1/\sqrt{2}$	0
$1/\sqrt{2}$	3.12	$-1/\sqrt{2}$	-3.12	0
0	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	0
0	0	0	0	1

Image Rotation Test Case



Unperturbed Matrix

0	0	1	10	0
0	0	0	1	0
-1	-10	0	0	0
0	-1	0	0	0
0	0	0	0	1

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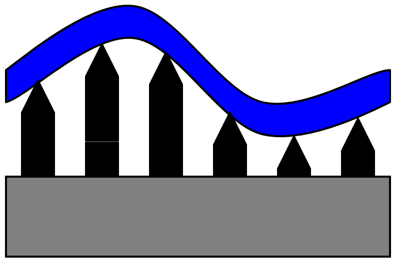
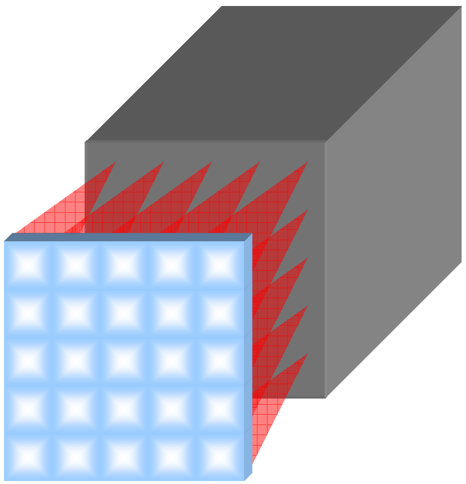
Rectifying the Wave-Optics Results with 5x5 Ray Matrix Effects

Rationale for Combining Ray and Wave Results (Rectification)

- In some locations, wave-optics and ray matrix effects need to be combined.
  - We call this process “rectification”.
- Example locations include
  - Deformable Mirrors
  - Aberrated Optics
  - Wavefront Sensors





Rectification Outline

- Ray Matrix Effects needing Rectification
- Effect Magnitude Determination
- Order of Operations
- Method of Applying Effects

Effects Modeled by Ray Matrix

- Reflection Inversion
- Image Rotation

Cannot be done with wave-optics.

- Magnification
- Power

Traditionally done with wave-optics, but can adversely impact wave-optics mesh parameters

- Translation
- Tilt

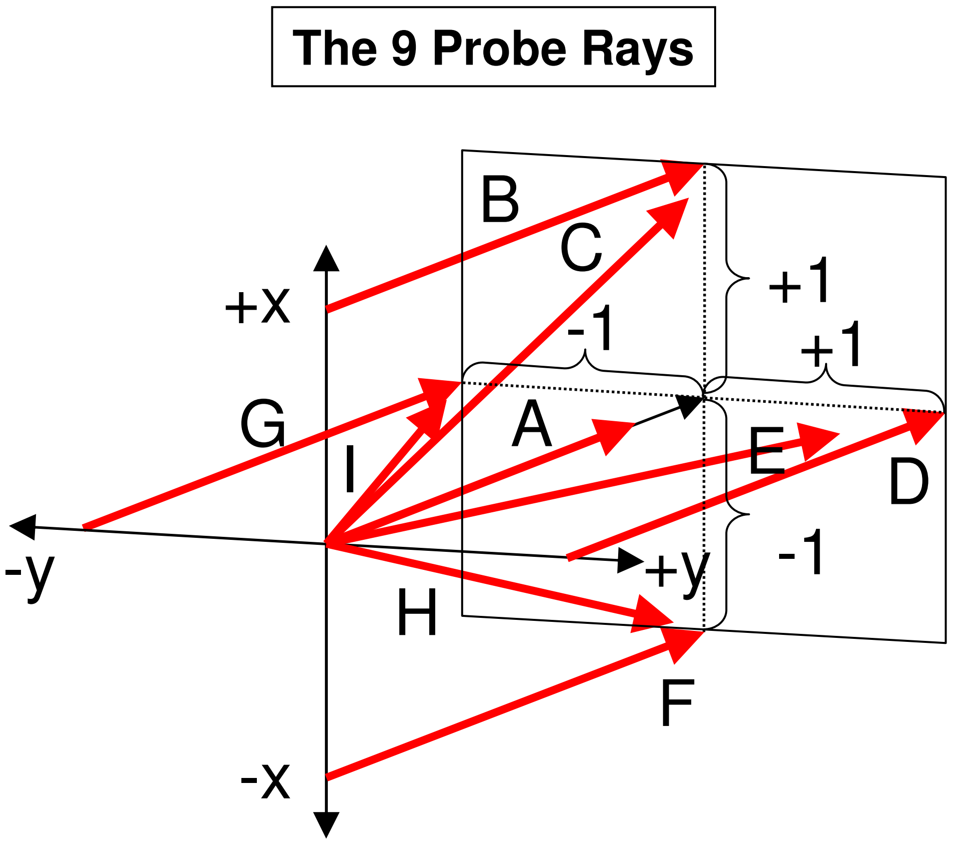
Can be done with wave-optics, but adversely affects the wave-optics mesh parameters

- Propagation

Needs to be done with wave-optics

Magnitudes of the Ray Effects

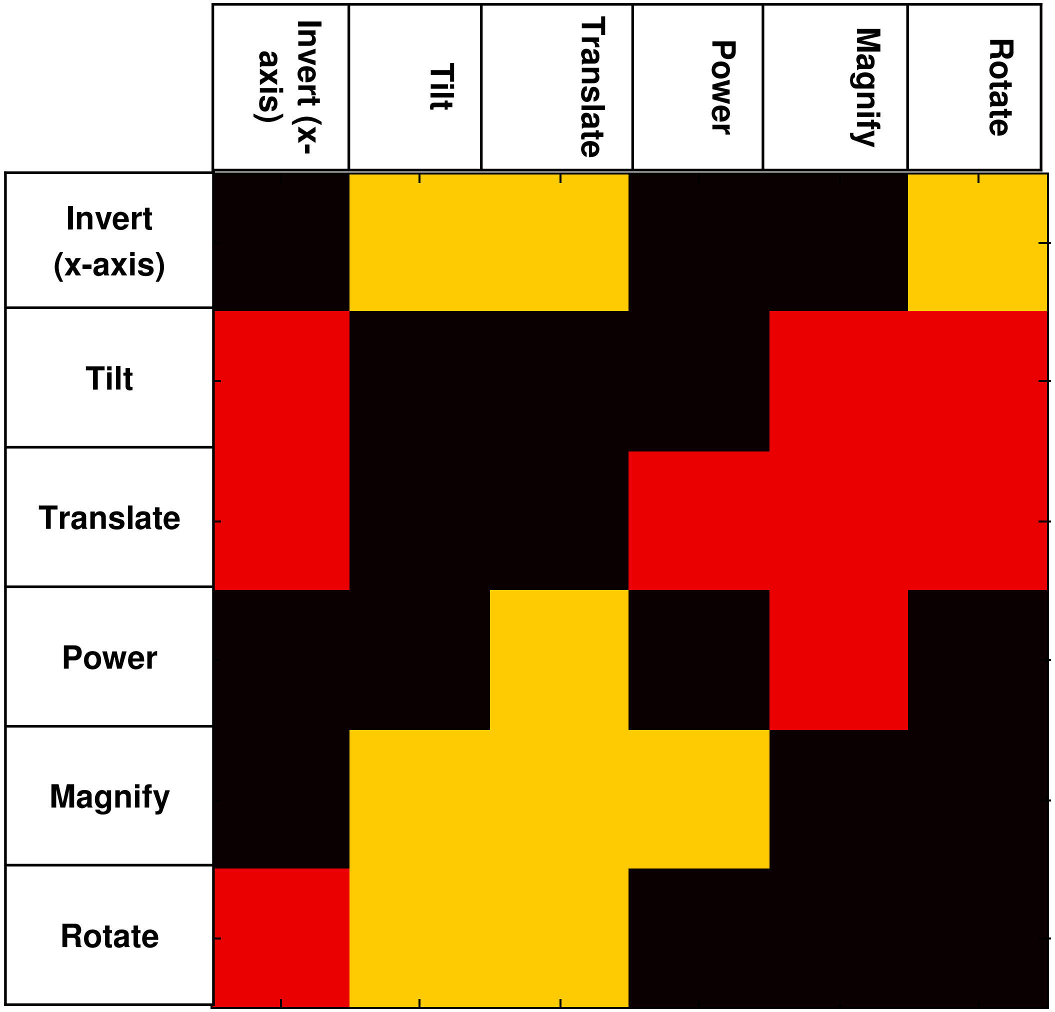
- Send 9 probe rays through the ray matrix
  - 4 angles, 4 offsets, and 1 center
- Analysis of each of the rays coming out of the system allows determination of the magnitudes of the effects



Rationale for Needing Order of Operations

- At a rectification plane, many ray matrix effects have been combined into a single ray-matrix
  - The order of application of the effects has been lost
- Operations happen in parallel
- Sometimes we only have the ray matrix
- Large number of sequential operations
  - Any 5x5 ray matrix can be decomposed into 7 effects
  - Minimizing the number of rectifications reduces noise
- Effects interact
  - EXAMPLE: power and magnification - applying power then magnifying reduces the optical power by the magnification

Operations Interaction Matrix



Key

- No Problem
- Column before Row
- Row before Column
- Potential Problem

This interaction matrix gives 15 possible orders.

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The 15 Potential Orders

Order	Operations					
1	invert	rotate	magnify	power	translate	tilt
2	**magnify**	**invert**	**rotate**	**power**	**translate**	**tilt**
3	invert	magnify	rotate	power	translate	tilt
4	magnify	power	invert	rotate	translate	tilt
5	magnify	invert	power	rotate	translate	tilt
6	invert	magnify	power	rotate	translate	tilt
7	invert	rotate	magnify	power	tilt	translate
8	magnify	invert	rotate	power	tilt	translate
9	invert	magnify	rotate	power	tilt	translate
10	magnify	power	invert	rotate	tilt	translate
11	magnify	invert	power	rotate	tilt	translate
12	invert	magnify	power	rotate	tilt	translate
13	invert	rotate	magnify	tilt	power	translate
14	invert	magnify	rotate	tilt	power	translate
15	invert	magnify	rotate	tilt	power	translate

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Rectification Implementation

- Two Options
  - Interpolate the complex field
  - Interpolate the applied phase/magnitude or sensor grid
- Interpolation of complex numbers tends to create more noise than interpolating real numbers, so generally the latter is preferred.

Conclusions & Future Work

- Developed a 5x5 Ray Matrix Formalism
- Developed a Technique for Integrating the effects modeled by ray matrices with Wave-Optics
- Future Work:
  – Show how the 5x5 ray matrix can be used to specify the wave-optics propagation

$$U_2(x_2,y_2)=\frac{\exp(jkL)}{j\lambda B}\int\int_{-\infty}^{\infty}U_1(x_1,y_1)_1$$

$$\exp\left[\frac{jk}{2B}\left(A\left(x_1{}^2+y_1{}^2\right)-2\left(x_1x_2+y_1y_2\right)+D\left(x_2{}^2+y_2{}^2\right)\right)\right]dx_1dy$$

From Siegman's *Lasers*

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Acknowledgements

- Thanks to the ABL SPO for their continued support of this development.
  - Special thanks to Dr. Salvatore Cusumano and Capt. Jason Tellez.

Questions?