MZA Associates Corporation

Scaling Law Modeling of Thermal Blooming in Wave Optics

Amy M. Ngwele, Associate Scientist
Matthew R. Whiteley, PhD, Vice President

November 2, 2006

DEPS Directed Energy Symposium
Albuquerque, NM

MZA Associates Corporation
6651 Centerville Business Pkwy Ste B
Dayton, OH 45459-2678
Voice: (937) 432-6560 x239

CLEARED
FOR PUBLIC RELEASE
AFRL/DED – PA
31 Oct 2006
AFRL/DE 06-578

Abstract

- **Relationship between peak irradiance reduction (Strehl) and distortion number for uniform beam. Scaling law previously anchored for Gaussian.**
  - **Overview of Thermal Blooming**
  - **Define Strehl**
  - **Difference between uniform and Gaussian**
  - **Data and Results**

- **Found dependence of distortion number on the path**

Thermal Blooming

- **Absorption of a high energy laser propagating in atmosphere causes heating of the air around the beam, changing it's optical properties (refractive index)**
- **The change in refractive index induces a phase variation across the beam resulting in spread (blooming) and a crescent shape**
- **The phase is typically characterized by distortion number**

$$\boxed{\begin{aligned} N_d &= \frac{4\sqrt{2}k_0}{\rho_0\, C_p}\int_0^L dz\, \frac{n_T(z)\alpha(z)}{V(z)D(z)} \\ &\quad \times P\int_0^z dz'\ \exp\left\{-[\alpha(z') + s(z')]\right\} \end{aligned}}$$

$$\begin{aligned} P &= \text{laser power} \\ k_0 &= \text{wavenumber, } 2\pi/\lambda \\ \rho_0 &= \text{density of air at sea level} \\ C_p &= \text{heat capacity at constant pressure} \\ n_T &= dn/dT = (n_0 - 1)/T \\ L &= \text{propagation path length} \\ V &= \text{beam clearing speed} \perp \text{to propagation} \\ D &= \text{beam diameter along path} \\ \alpha &= \text{absorption coefficient along path} \\ s &= \text{scattering coefficient along path} \end{aligned}$$

- **Can derive relationship between distortion number and peak irradiance reduction**

$$S = (1 + f(N_d))^{-1}$$

MZA Associates Corporation

Path Effects on N_d

- **Absorption profile, $\alpha(z)$, varies with scenario**
- **Attenuation of power along path**

$$P(z) = P\int_0^z dz' \ \exp\left\{-[\alpha(z') + s(z')]\right\}$$

- **Beam clearing speed, $V(z)$, varies with scenario**
- **Beam size, $D(z)$, is only parameter for which we have to make an assumption**

$$g_1 = \frac{D}{2}\left(1 - \frac{z}{f}\right),\ g_2 = \frac{4z}{k_0 D}$$

$$D(z) = 2\sqrt{g_1^2 + g_2^2}$$

Thermal Blooming Strehl

- **Define Strehl as ratio of peak irradiance with blooming (wherever it occurs) to peak irradiance in vacuum with the same power on target**

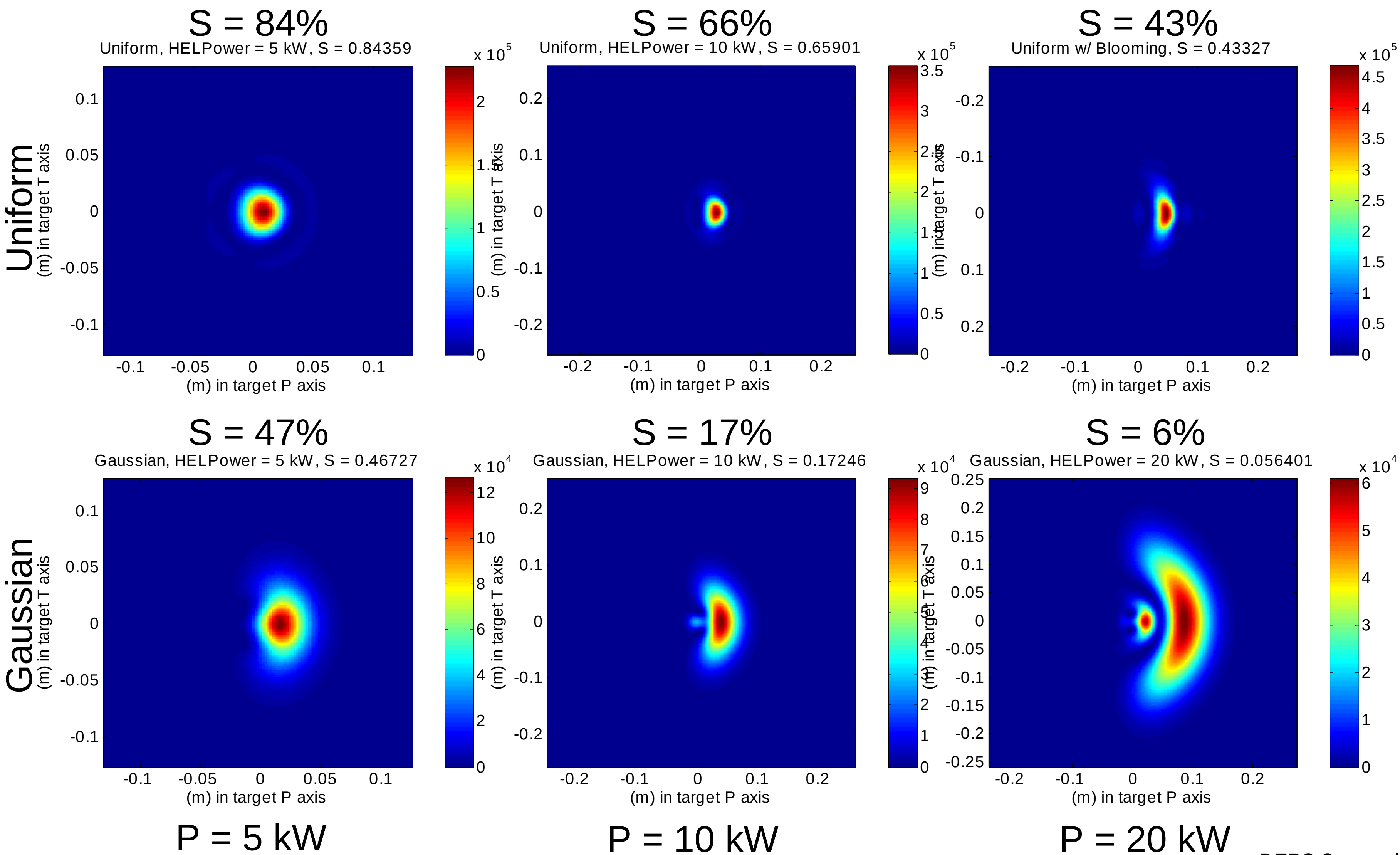
$$S = \frac{I_{pk}}{I_{pk_{vac}}}$$

- **Written another way**

$$\begin{aligned} P &= I_{pk} \int dx \int dy \; \frac{I(x,y)}{I_{pk}} \\ &= I_{pk} A = I_{pk_{vac}} A_{vac} \text{ (energy conservation)} \end{aligned} \qquad A \equiv \int dx \int dy \; \frac{I(x,y)}{I_{pk}}$$

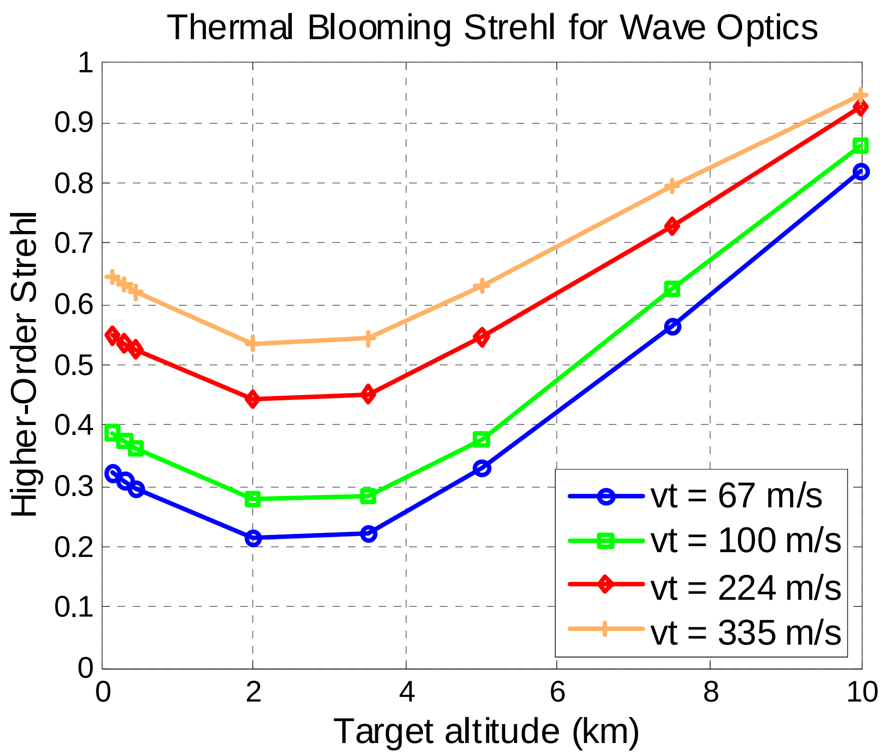
$$\begin{aligned} \rightarrow S &= \frac{A_{vac}}{A} = \frac{\sigma_{X_{vac}} \sigma_{Y_{vac}}}{\sigma_X \sigma_Y} \\ &\boxed{= \left[\left(\frac{\sigma_X}{\sigma_{X_{vac}}} \right) \left(\frac{\sigma_Y}{\sigma_{Y_{vac}}} \right) \right]^{-1}} \end{aligned}$$

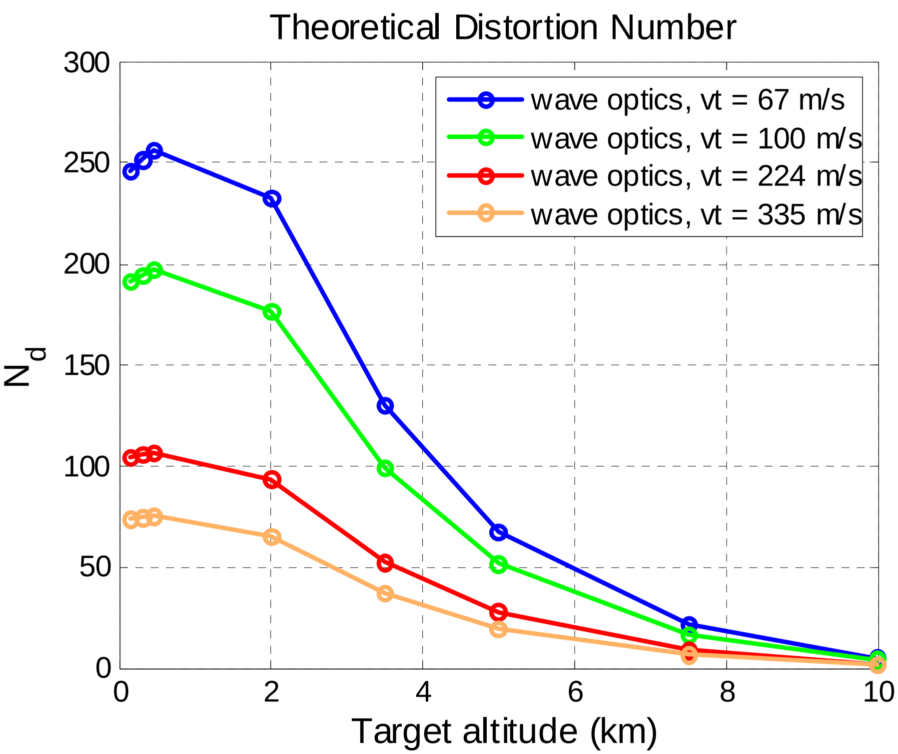
Uniform vs. Gaussian



Motivation for Path Weighting – Strategic Scenario

MZA Associates Corporation



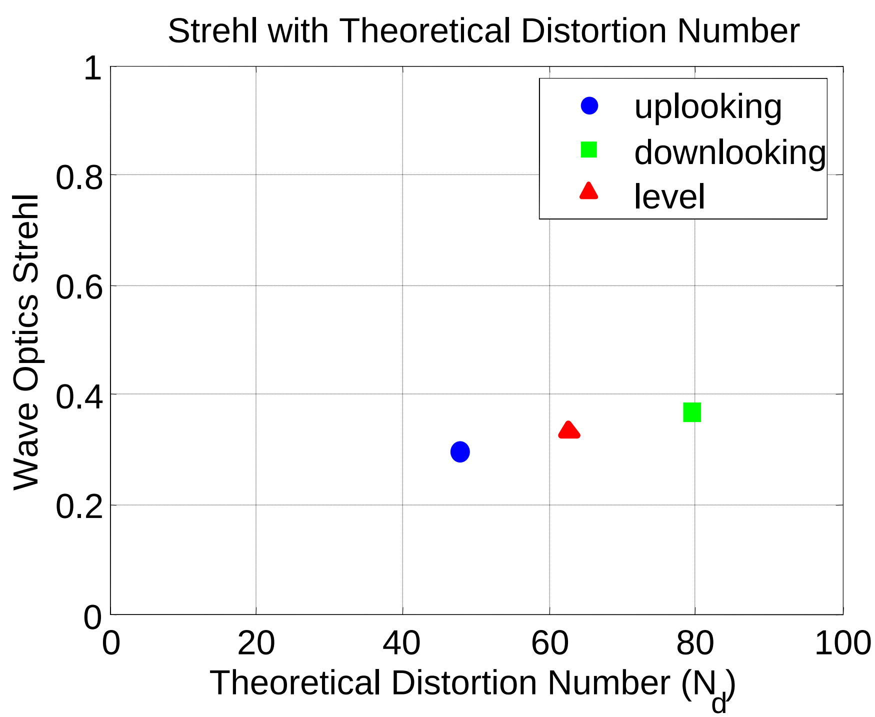


- **Wave optics yields approximately same Strehl for 0 and 5 km altitude targets**
- **Theoretical distortion number calculation gives very different results for 0 and 5 km altitude targets, factor of 3 difference**
- **Scaling law is overestimating the effect for very low altitude targets**

MZA Associates Corporation

Motivation for Path Weighting – Tactical Scenario

- **Ran wave optics simulation for three generic geometries with 15 km path length from platform to target**
  - **Uplooking:**
    - **Platform alt – 1 m**
    - **Target alt – 4000 m**
  - **Downlooking:**
    - **Platform alt – 4000 m**
    - **Target alt – 1 m**
  - **Level:**
    - **Platform alt – 1500 m**
    - **Target alt – 1500 m**
- **Atmosphere**
  - **Absorption/Scattering – follow logarithmic scaling**
  - **Uniform natural wind, 10 m/s, transverse to beam path**

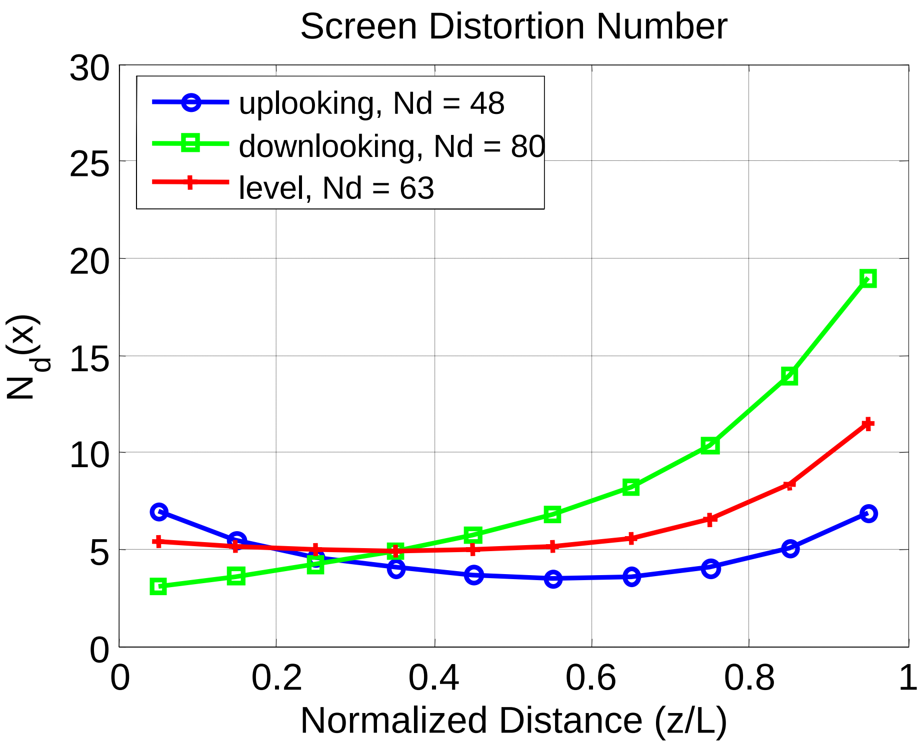


Path Dependence of Distortion Number

MZA Associates Corporation

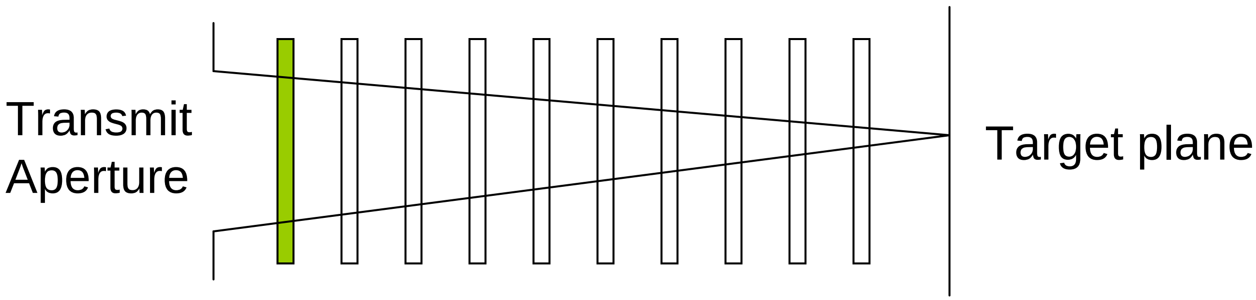
- **Calculated distortion numbers for a down-looking geometry typically overestimate the effect of blooming while calculation for up-looking geometries underestimate.**

- **This motivates a path weighting of the distortion number.**



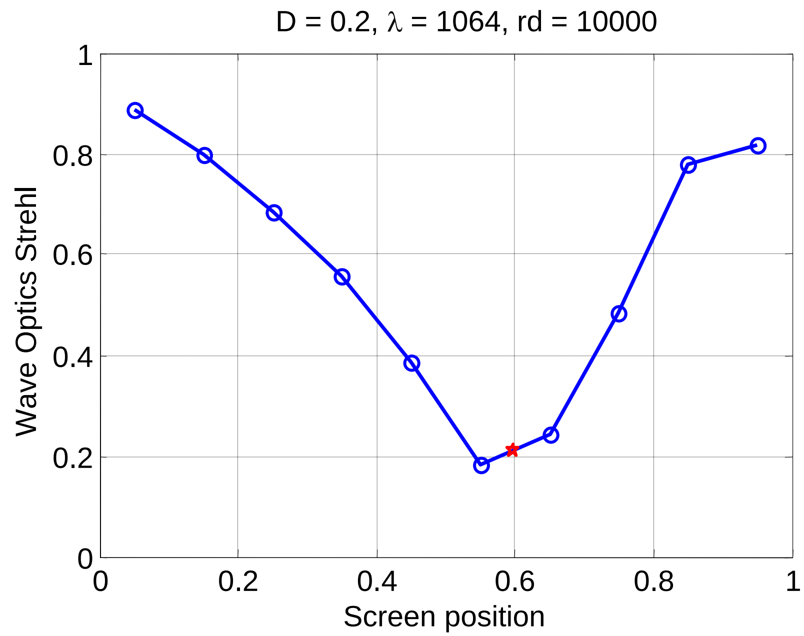
MZA Associates Corporation

Path Weighting Determination



Green screen is absorbing screen

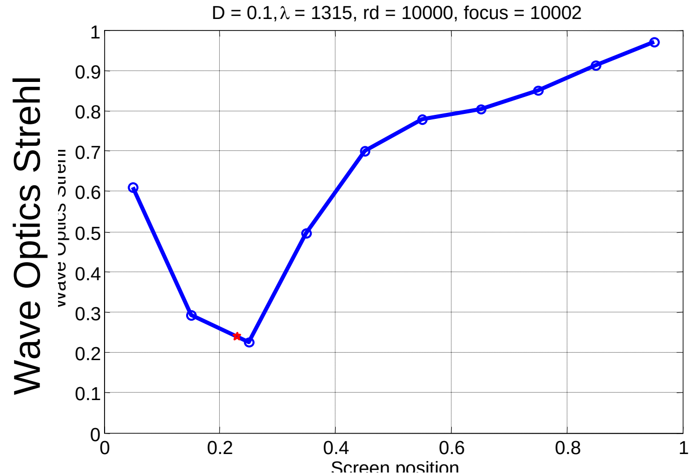
- **Waveoptics runs with uniform extinction. One screen is an absorbing screen and is moved along the path toward the target.**
- **There is a point in the path at which the distortion reaches a maximum, and after which the distortion decreases as the blooming screen is moved further toward the target.**



MZA Associates Corporation

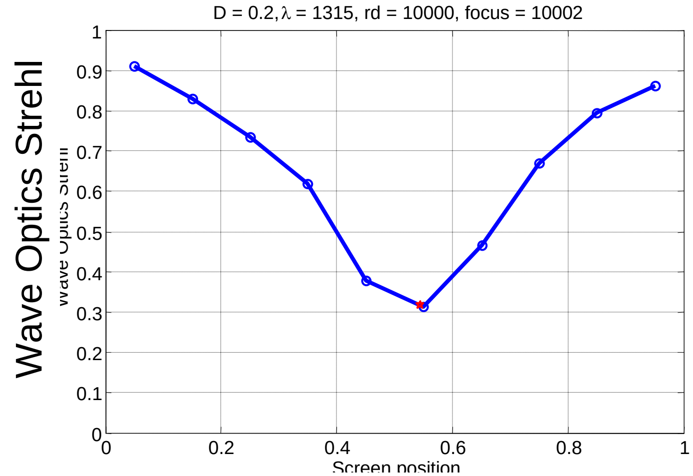
Strehls for single screen runs

D = 10 cm



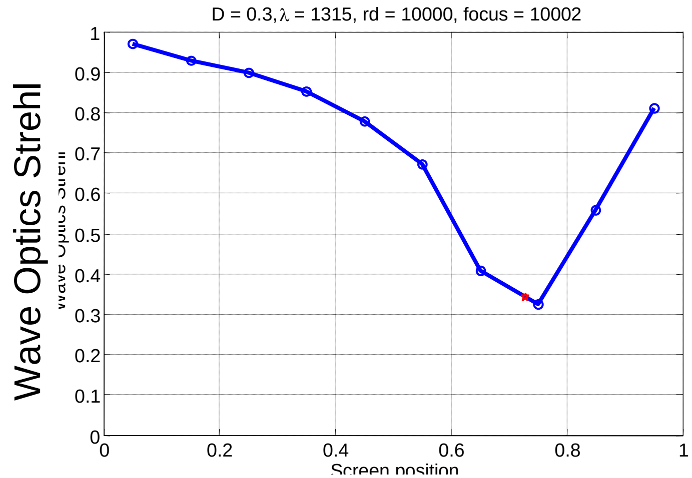
Screen Position (z/L)

D = 20 cm



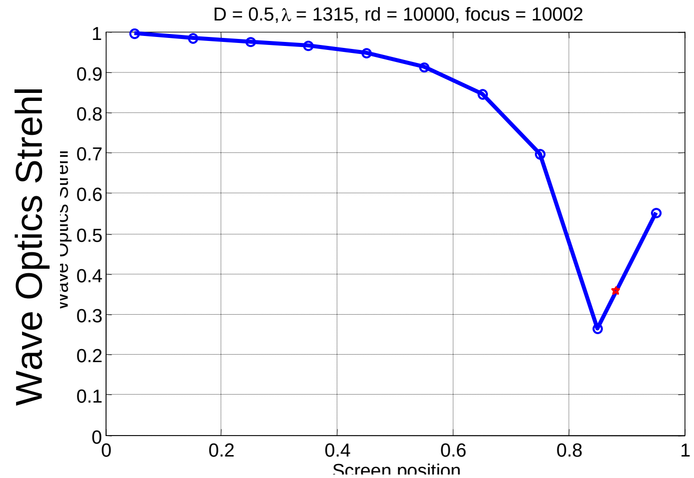
Screen Position (z/L)

D = 30 cm



Screen Position (z/L)

D = 50 cm



Screen Position (z/L)

MZA Associates Corporation

Point of Maximum Distortion

- **Point of maximum distortion was found to correspond to the point in the path where the size of the beam in vacuum would start to be affected by diffraction**

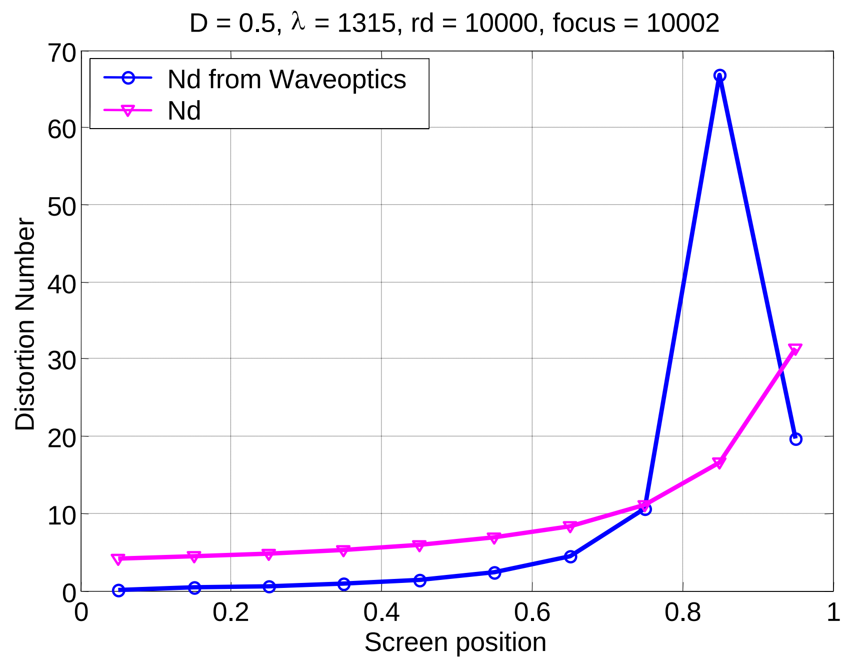
$$g_1 = \frac{D}{2}\left(1 - \frac{z}{f}\right),\ g_2 = \frac{4z}{k_0 D}$$

- **Fresnel number dependence on position of maximum distortion, $g_2 = 0.5\ g_1$**

$$x_{pk} = \frac{N_f}{4 + N_f},\ \text{where } N_f = \frac{2\pi}{\lambda}\frac{D^2}{4L}$$

Wave-Optics Distortion Number

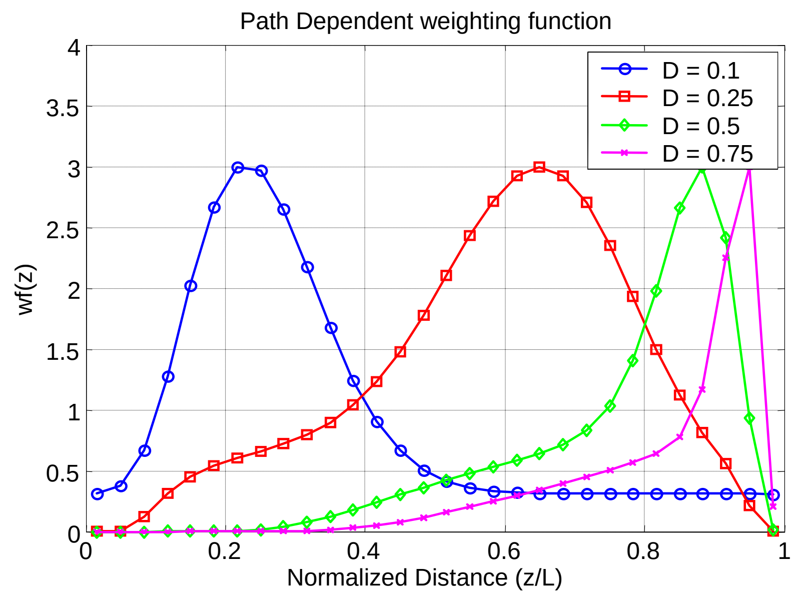
- Using wave-optics Strehl in fit to level path data for Strehl vs. N_d
- Normalize by N_d to get form of path weighting



Distortion Number Path Weighting

- **Peak moves toward platform with smaller aperture.**
- **Maximum of path weighting approximately constant = 3**

$$w(x) = \frac{3\,w(x)}{w(x_{pk})}$$



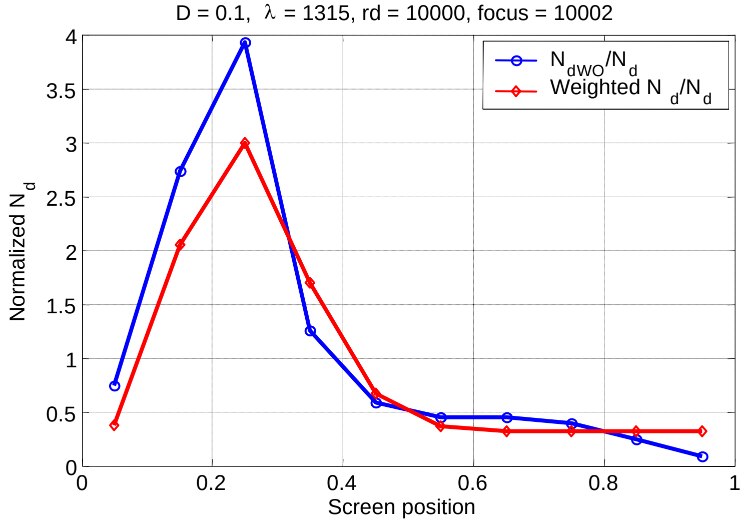
$$w(x) = \begin{cases} \sin\left(\pi(1-x)^{-\log(2)/\log(1-x_{pk})}\right)^4 + w_{tails}(x) & x_{pk} < 0.5 \\ \sin\left(\pi x^{-\log(2)/\log(x_{pk})}\right)^4 + w_{tails}(x) & x_{pk} \geq 0.5 \end{cases}$$

$$w_{tails}(x) = 0.5 x_{pk} \exp\left(-\left(\frac{x - x_{pk}}{x(1-x)/(x_{pk}^4)}\right)^2\right)$$

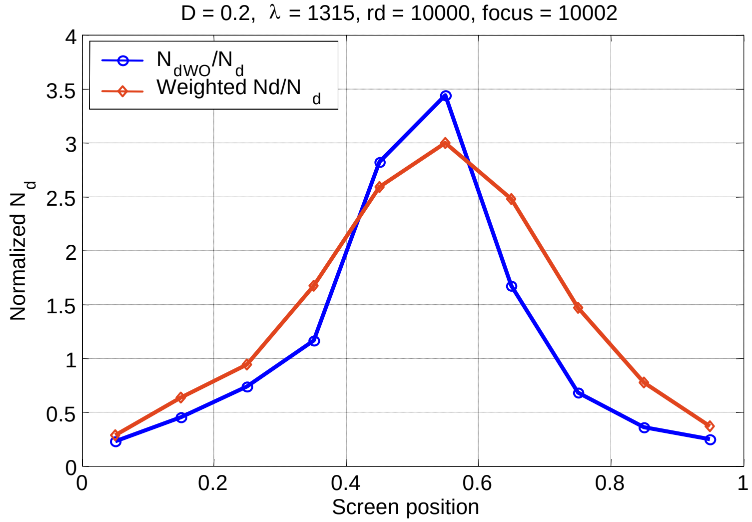
MZA Associates Corporation

Normalized distortion parameter, single screen runs

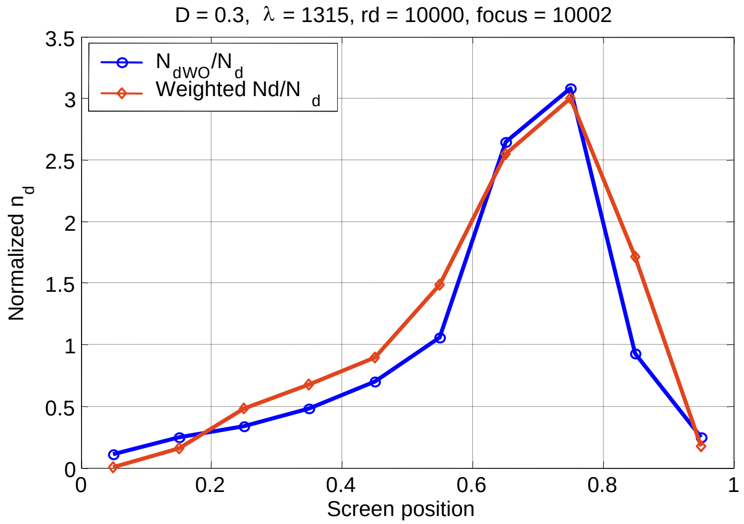
D = 10 cm



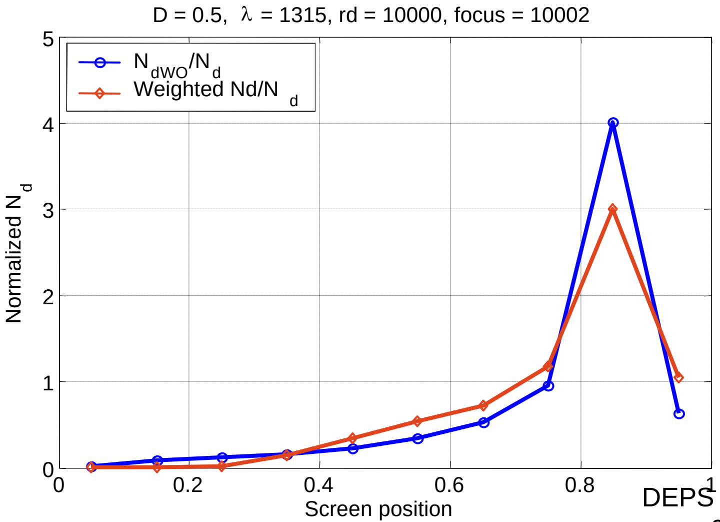
D = 20 cm



D = 30 cm



D = 50 cm



Transmission Scaling

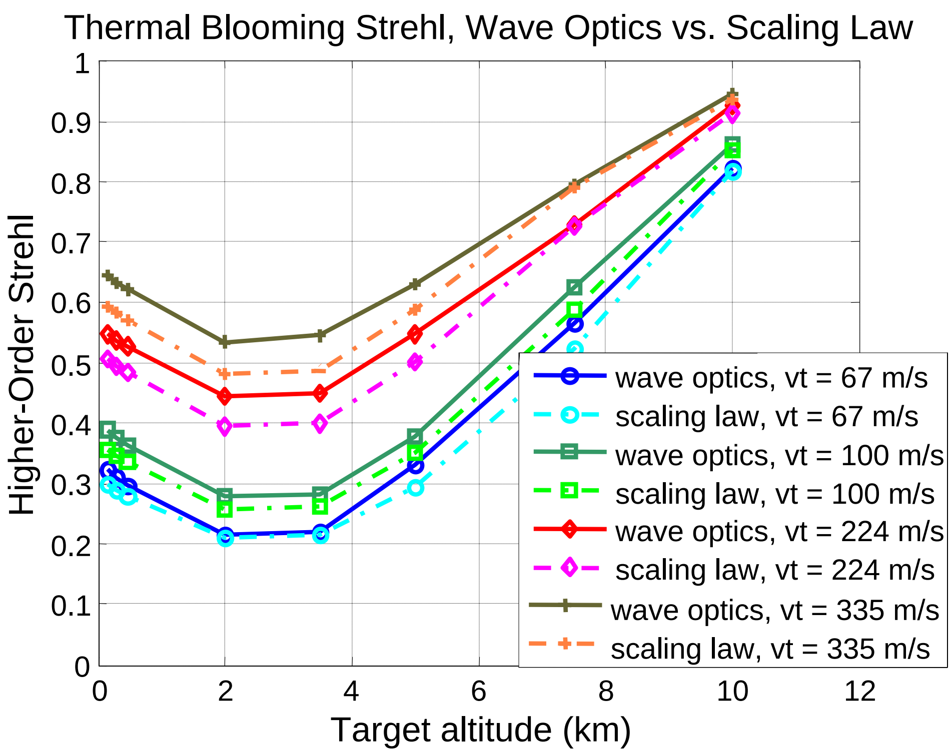
- **To bring uplooking and downlooking geometries together, introduced transmission scaling, γ**

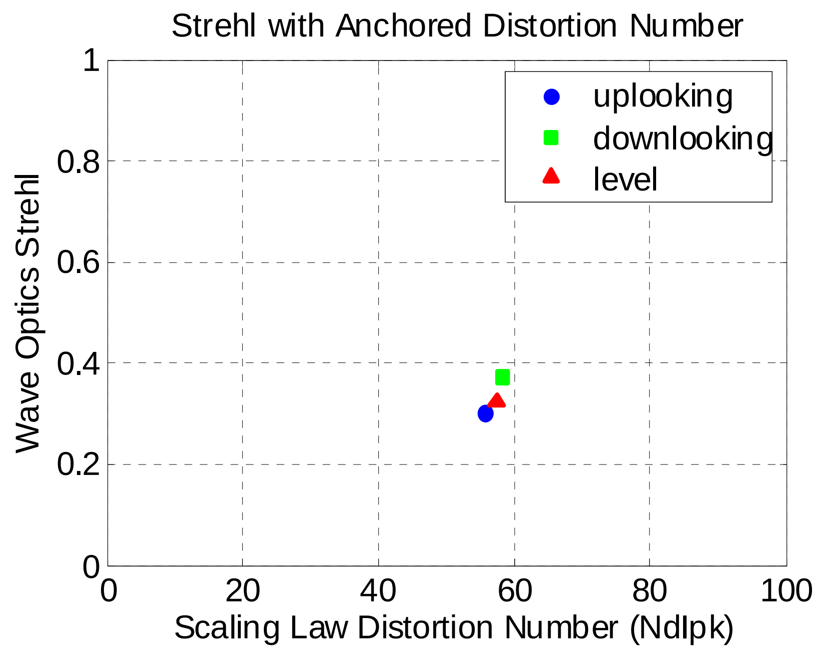
$$\gamma_0 = T_{Scat}^{\frac{\alpha_N - \alpha_1}{2\bar{\alpha}}}$$

$$\gamma = \begin{cases} \gamma_0 - 0.1\left(1 - e^{2(0.4-\gamma_0)}\right), & \gamma_0 < 0.4 \\ \gamma_0, & \gamma_0 \geq 0.4 \end{cases}$$

$$\begin{aligned} \gamma_0 &= \text{Initial calculation of transmission scaling} \\ T_{Scat} &= \text{Transmission due to scattering over entire path} \\ \alpha_i &= \text{Extinction coeffcient for the } i\text{th segment} \\ \bar{\alpha} &= \text{Mean extinction coefficient} \end{aligned}$$

Final Results





$$N_{dIpk} = \gamma \int_0^L w(z) N_d'(z) dz$$

where $N_d'(z)$ is the integrand in equation for Nd, then

Remarks

- Starting from the Bradley Herrmann definition of distortion number, the only assumption made is beam size
- Beam size at some point in the path is computed as the combination of the effects of focus and diffraction based on Gaussian beam assumptions, and does not include any effects due to thermal blooming prior to that point in the path
- The transmission scaling factor can be thought of as a modification to the computed beam size to account for changes due to thermal blooming in the path

$$N_{dIpk} = \int_0^L w(z) \frac{N_d'(z) D(z)}{D_{eff}(z)} dz$$

$$\text{where } D_{eff}(z) = \frac{D(z)}{\gamma}$$

Conclusion

- **Path weighting and transmission scaling of distortion number - excellent agreement with wave optics**
- **Anchoring with tactical and strategic engagements**

- **Reference**

L. C. Bradley and J. Hermann, "Phase Compensation for Thermal Blooming", Appl. Opt., vol. 13, pp. 331-334, 1974.