Determining Wave-Optics Mesh Parameters for Modeling Complex Systems of Simple Optics

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Outline

- Introduction & Motivation
- Aperture Imaging Into Input Space
- Finding the Field Stop & Aperture Stop
- Mesh Determination for Complex Systems
- Conclusions

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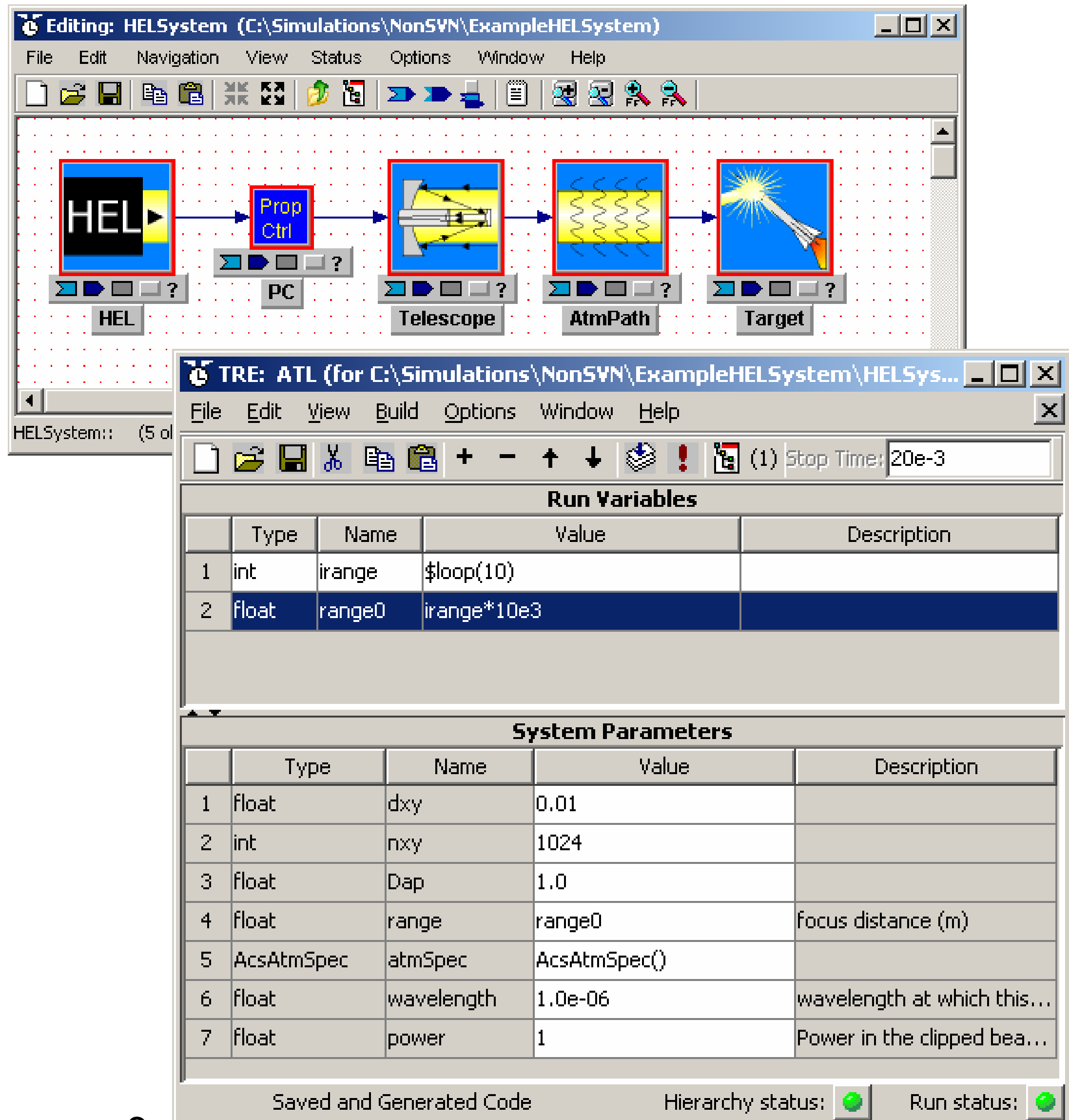
Introduction to WaveTrain

- WaveTrain
  - Wave-Optics Modeling Tool Based on tempus
  - FREE for government work
- WaveTrain is becoming the industry standard for wave-optics modeling
- An investment in WaveTrain or tempus is not lost because for government use they are:
  - open-source & non-proprietary
- tempus can work with existing modeling software.
  - no duplication of effort or need to learn too much new software



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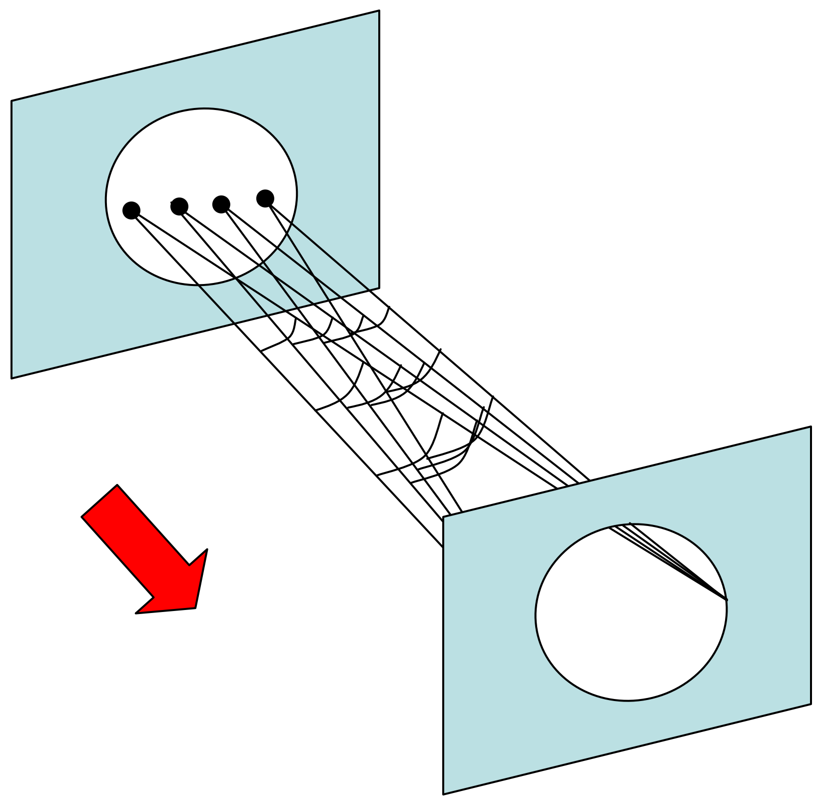
Huygens Principle

In 1678 Christian Huygens "expressed an intuitive conviction that if each point on the wavefront of a light disturbance were considered to be a new source of a secondary spherical disturbance, then the wavefront at any later instant could be found by construction the envelope of the secondary wavelets."

-J. Goodman, *Introduction to Fourier Optics* (McGraw Hill, 1968), p. 31.



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Simple Fourier Propagator & Notation Simplification

$$U_2 = P \cdot \iint U_1 \cdot h \cdot dx_1 \cdot dy_1, \quad h = \exp\left(j \frac{k}{2z} \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 \right] \right)$$

$$Q_1 = \exp\left(jk \frac{r_1^{\,2}}{2z} \right)$$

Quadratic Phase Factor (QPF): Equivalent to the effect a lens has on the wavefront of a field.

$$F(U) = \iint U \cdot \exp\left(-j \frac{k}{z} (x_2 x_1 + y_2 y_1) \right) \cdot dx_1 \cdot dy_1$$

Fourier Transform

$$P = \frac{\exp(jkz)}{jkz}$$

Multiplicative Phase Factor: Takes into account the overall phase shift due to propagation

$$U' = P \cdot Q_2 \cdot F(U \cdot Q_1)$$

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Convolution Propagator – Two FTs

- Steps:
  - Fourier transform
  - multiplication by the Fourier transformed kernel
  - an inverse Fourier transform
- Advantage:
  - Allows control of the mesh spacing
- Remaining Question for FFT Implementation:
  - What mesh size and spacing should be used?

$$U_2 = P \cdot \iint U_1 \cdot h \cdot dx_1 \cdot dy_1$$

$$h = \exp\left(j \frac{k}{2z} \left[\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 \right] \right)$$

$$F(h) = H = \exp\left[- j\pi\lambda z \left(f_x^{\ 2} + f_y^{\ 2} \right) \right]$$

$$U_2 = P \cdot F^{-1}\left(F(h) \cdot F(U_1) \right)$$
$$= P \cdot F^{-1}\left(H \cdot F(U_1) \right)$$

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Prior Work: Rules of Thumb

- Siegman gives guidance for single propagations as follows :
  - Number of samples "between 2N and 8N" where N is the Fresnel number and
  - Guard band of "≈1.2 to ≈ 1.5 times the half-width of the aperture itself." (*Lasers*, **18.3**)
- Another General Procedure:
  - Double the mesh size and reduce the spacing by √2 and see if the answer matches the lower resolution one.

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Picking Mesh Parameters for Simple Systems

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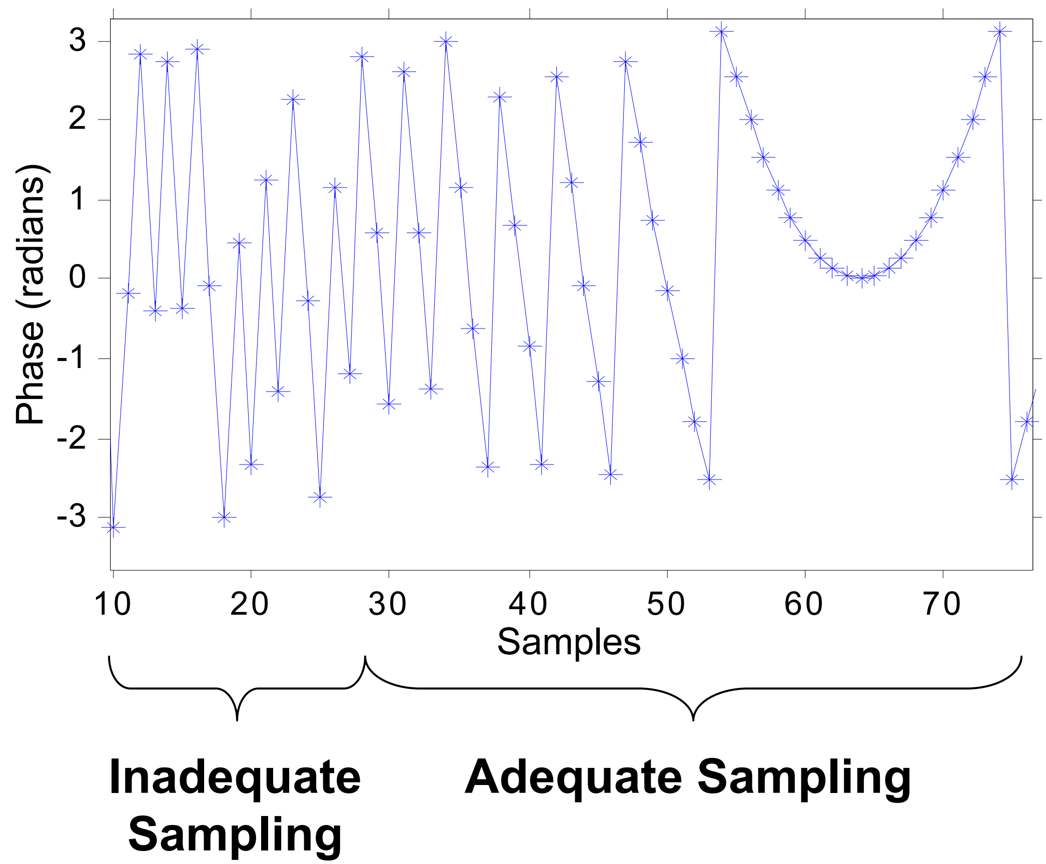
Adequate Phase Sampling

- In most situations, the most rapidly varying part of the field is the QPF.
- In a complex field, the phase is reset every wavelength or 2π radians.
- To achieve proper sampling, sampling theory dictates that we need two samples per wave.



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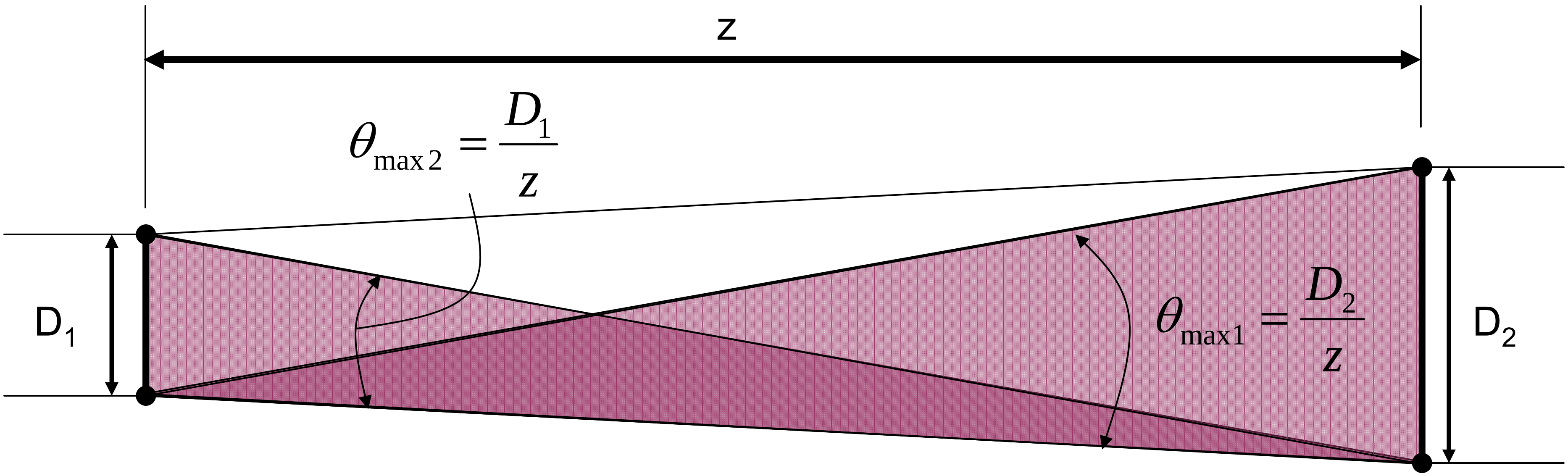
Mesh Sampling: Angular Bandwidth



Fully General Result

$$\delta_1 D_2 + \delta_2 D_1 \leq \lambda z$$

$$\frac{D_1}{z} \leq \frac{\lambda}{2\delta_2}$$

$$\delta_2 \leq \frac{z\lambda}{2D_1}$$

Simplified Result

$$\frac{D_2}{z} \leq \frac{\lambda}{2\delta_1}$$

$$\delta_1 \leq \frac{z\lambda}{2D_2}$$

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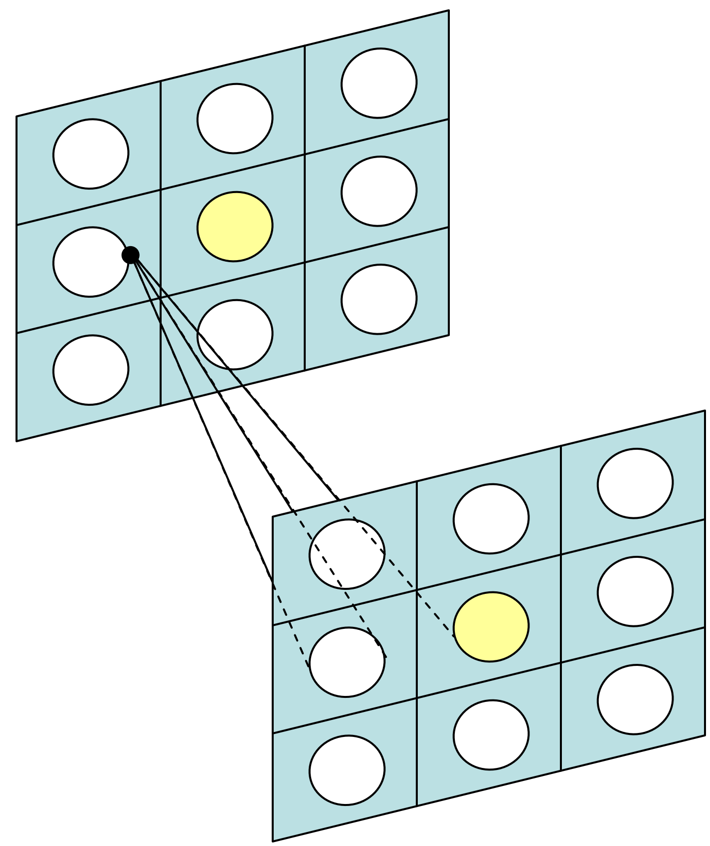
Virtual Adjacent Apertures – “Wrap Around”

- Now that we know the mesh sampling intervals (δ_1 and δ_2), we need to know how big a mesh we need to use to accurately model the diffraction.
- The Fourier transform assumes a repeating function at the input.
  - This means that there are effective virtual apertures on all sides of the input aperture.
- We need a mesh large enough that these virtual adjacent apertures do not illuminate our area of interest.
  - This allows us to avoid “wrap-around” by using a guard band.



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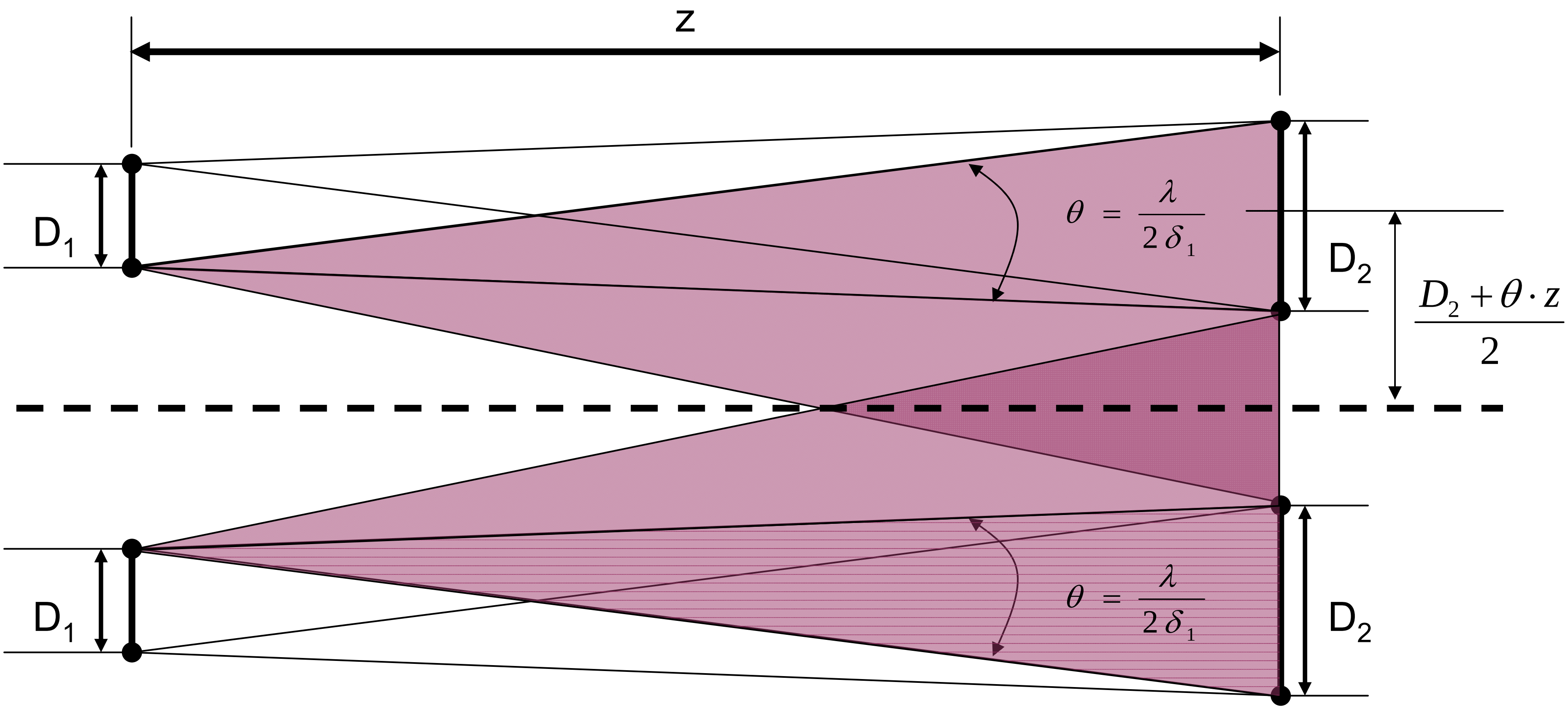
Mesh Size: Avoid "Wrap-Around"



Fully General Result

$$N \geq \frac{D_1}{2\delta_1} + \frac{D_2}{2\delta_2} + \frac{z\lambda}{2\delta_1\delta_2}$$

NOTE: Drawn for $\delta_1/\delta_2 = D_1/D_2$ and maximum mesh spacing.

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Mesh Determination Rules of Thumb

Mesh Sample Spacing

$$\delta_2 \le \frac{z\lambda}{2D_1} \text{ and } \delta_1 \le \frac{z\lambda}{2D_2}$$

Approximation: Mesh spacing should be bigger than half the diffraction limited radius from the other end.

Mesh Size

$$N \ge 16 N_{f\_eff} = 16 \frac{r_1 r_2}{\lambda z}$$

$$\text{for maximum } \delta_1 \text{ and } \delta_2$$

Approximation: Mesh size should be bigger than 16 times the effective Fresnel number.

S. Coy, "Choosing Mesh Spacings and Mesh Dimensions for Wave Optics Simulation" SPIE (2005).

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Determining Fourier Propagation Mesh Parameters for Complex Optical Systems of Simple Optics

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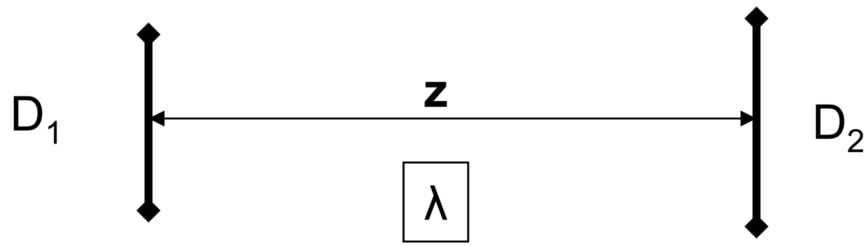
Introduction

- Wave-optic mesh parameters can be uniquely determined by a pair of limiting apertures separated by a finite distance and a wavelength.



- An optical system comprised of a set of ideal optics can be analyzed to determine the two limiting apertures that most restrict rays propagating through the system using field and aperture stop techniques.

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Definitions of Field & Aperture Stop

- **Aperture Stop** = the aperture in a system that limits the cone of energy from a point on the optical axis.
- **Field Stop** = the aperture that limits the angular extent of the light going through the system.
  - NOTE: All this analysis takes place with ray optics.

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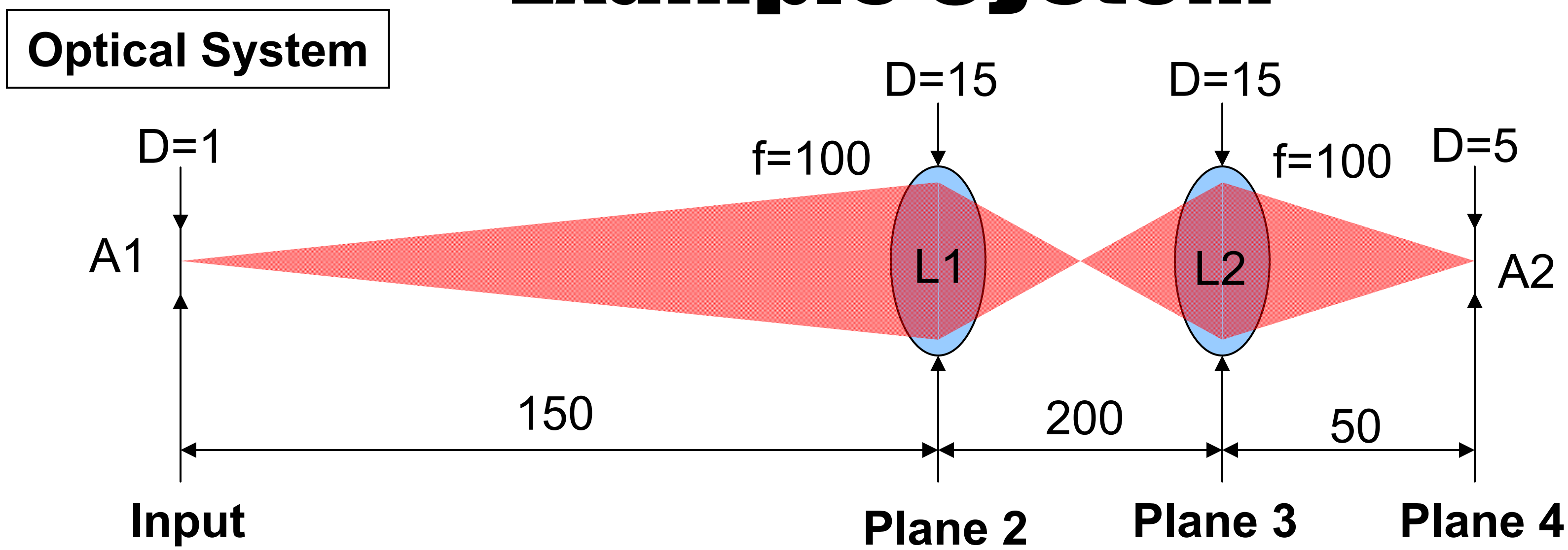
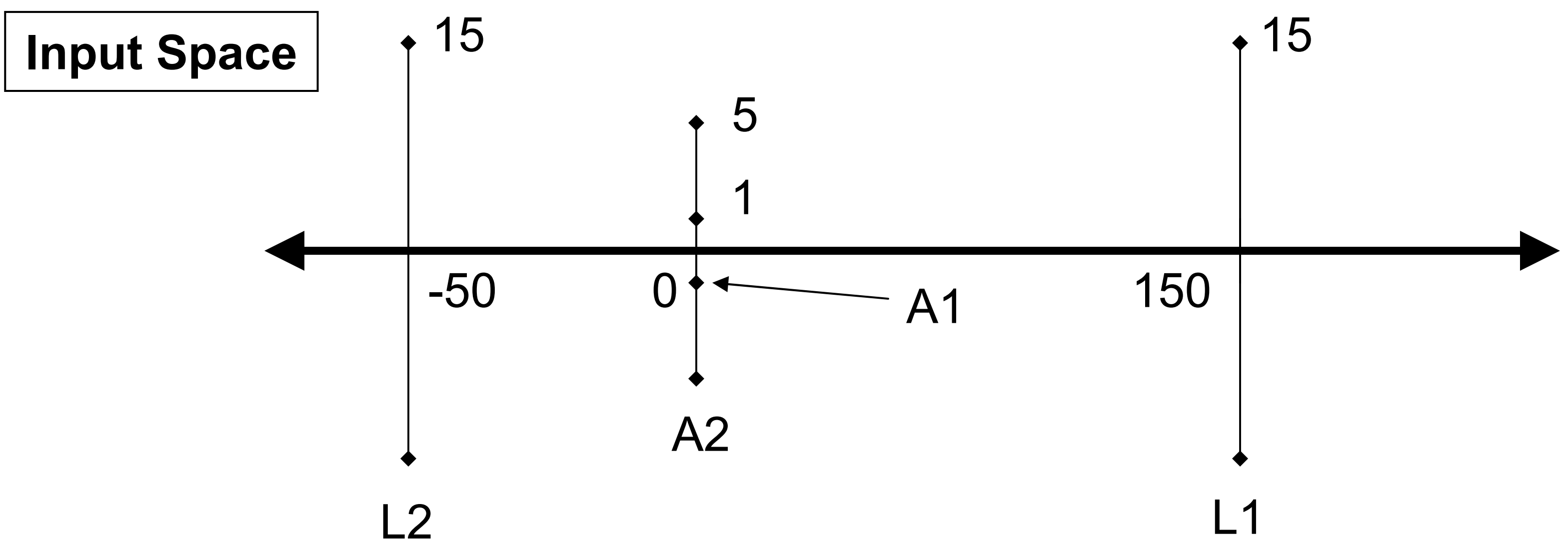
Example System





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Procedure for Finding Stops 1/3

Find the location and size of each aperture in input space.

1. Find the ABCD matrix from the input of the system to each optic in the system.
2. Solve for the distance (z_{image}) required to drive the B term to zero by inverting the input-space to aperture ray matrix.
   - This matrix is the mapping from the aperture back to input space.
3. The A term is the magnification (M_{image}) of the image of that aperture.

$$M_i = M_{input-space\_to\_aperture}^{-1} = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$$

$$\begin{bmatrix} 1 & z_{image} \\ 0 & 1 \end{bmatrix} \cdot \left(\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \right) = \begin{bmatrix} M & 0 \\ C & 1/M \end{bmatrix}$$

$$z_{image} = -B/D$$

$$M_{image} = C \cdot z_{image} + A$$

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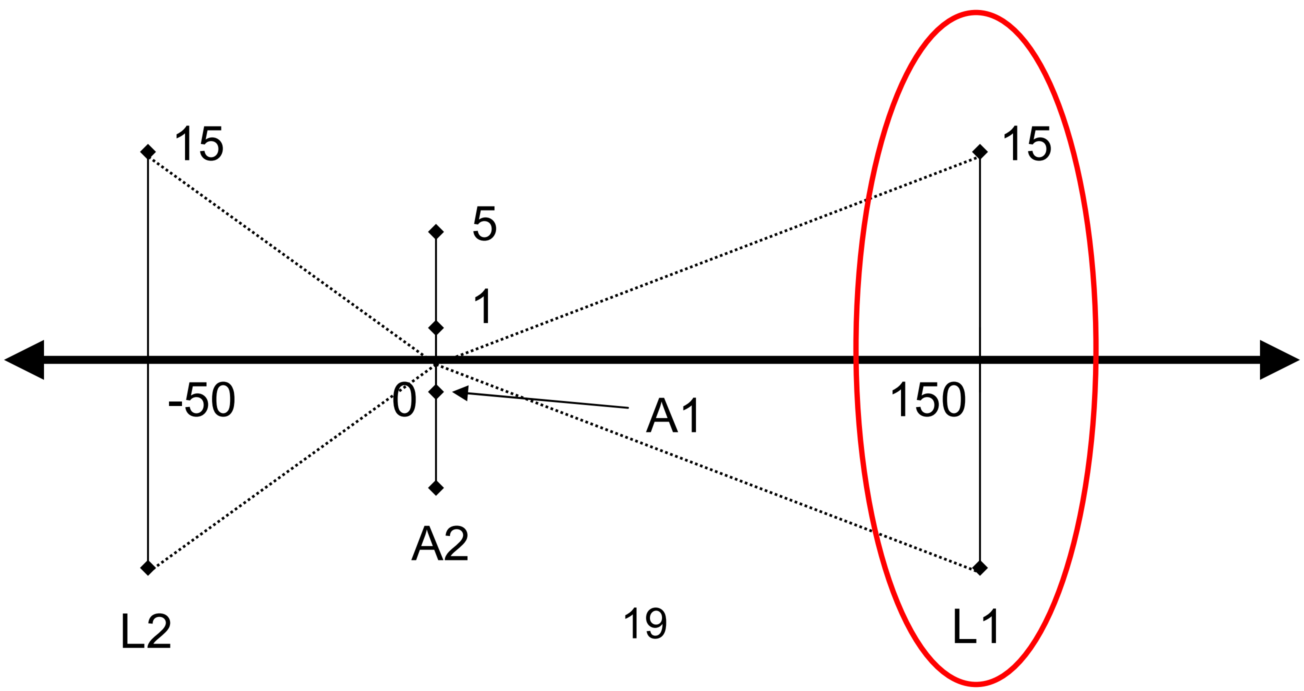
Procedure for Finding Stops 2/3

2. Find the angle formed by the edges of each of the apertures and a point in the middle of the object/input plane. The aperture which creates the smallest angle is the image of the aperture stop or the entrance pupil.



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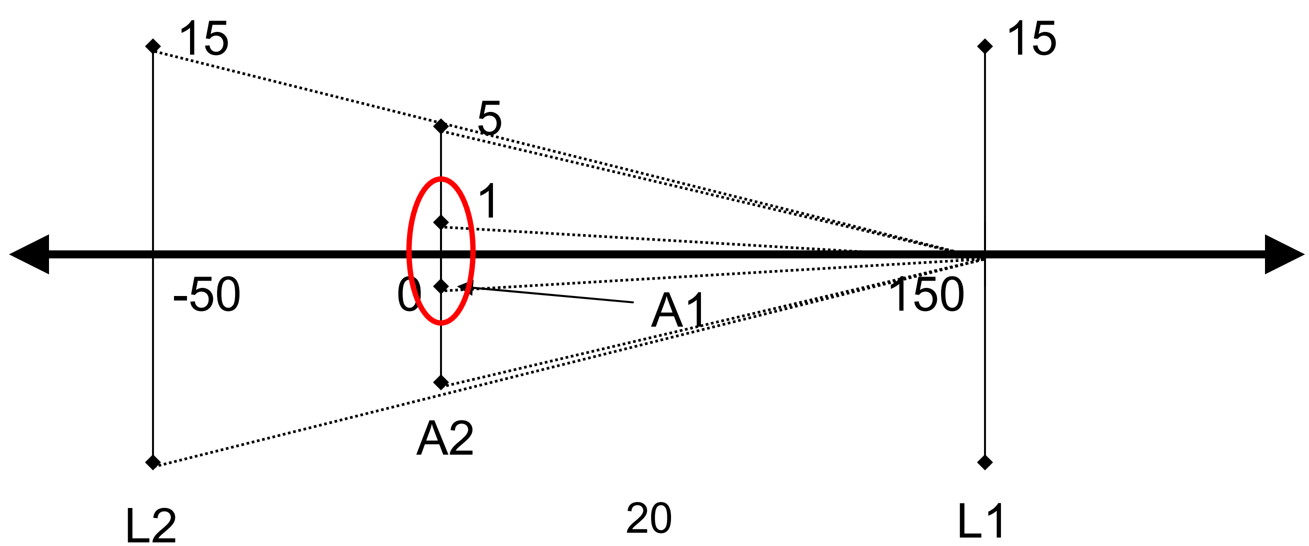
Procedure for Finding Stops 3/3

2. Find the aperture which most limits the angle from a point in the center of the image of the aperture stop in input space. This aperture is the field stop.



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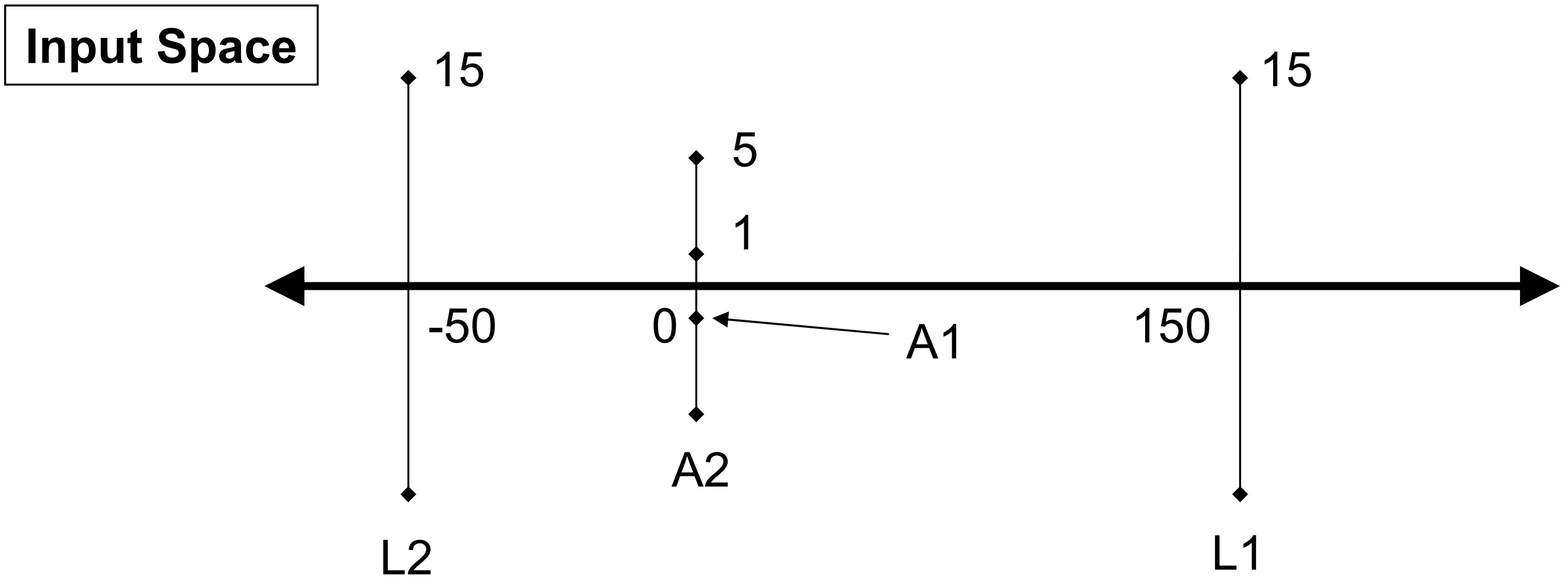
Example: Fourier Propagation



D1 = 1 mm, D2 = 15 mm, λ = 1 µm, z = 0.15 m

Minimal Mesh = 400 x 9.375 µm = 3.75 mm

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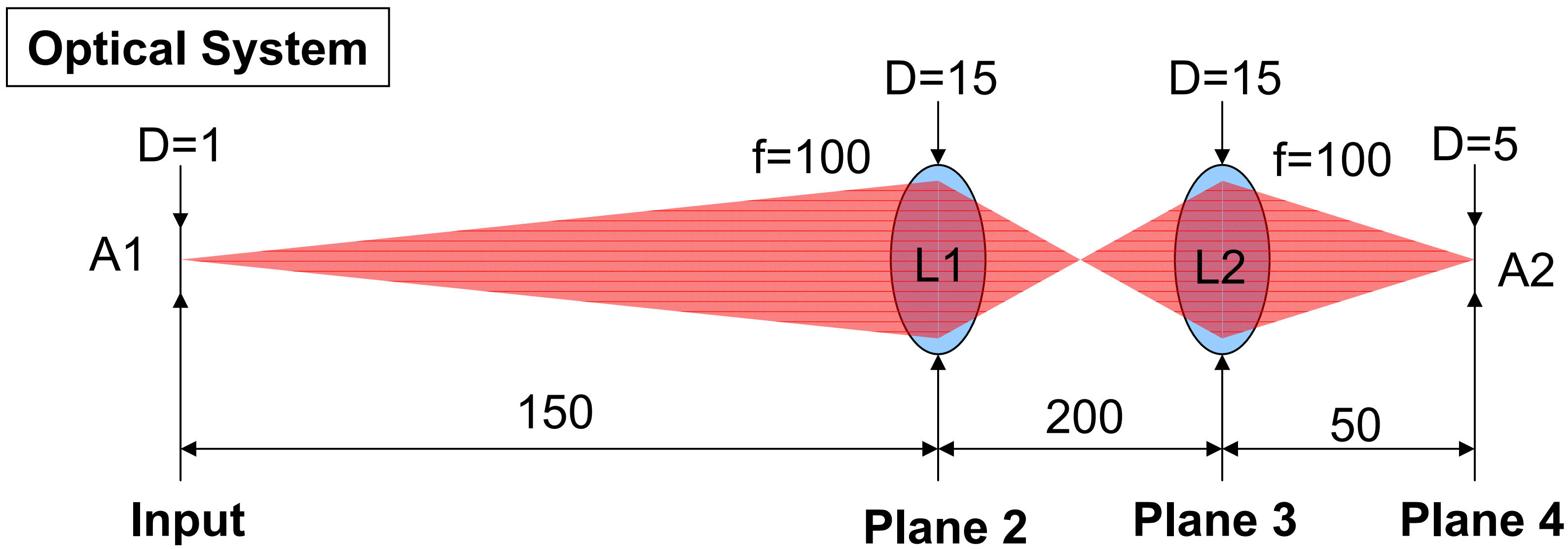
Example System Modeled



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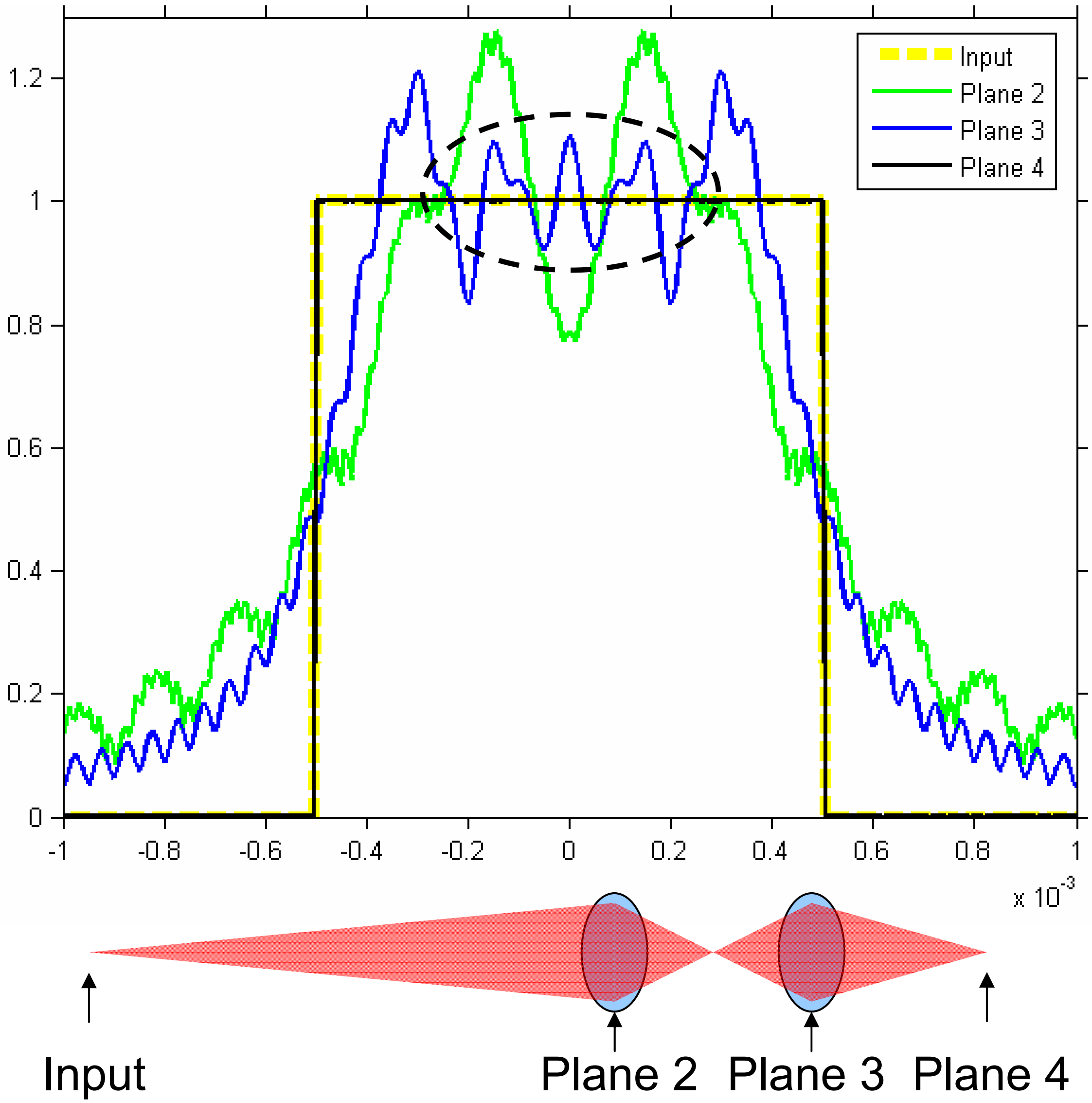
N=1024, δ=6.6 μm

Over-Sampled



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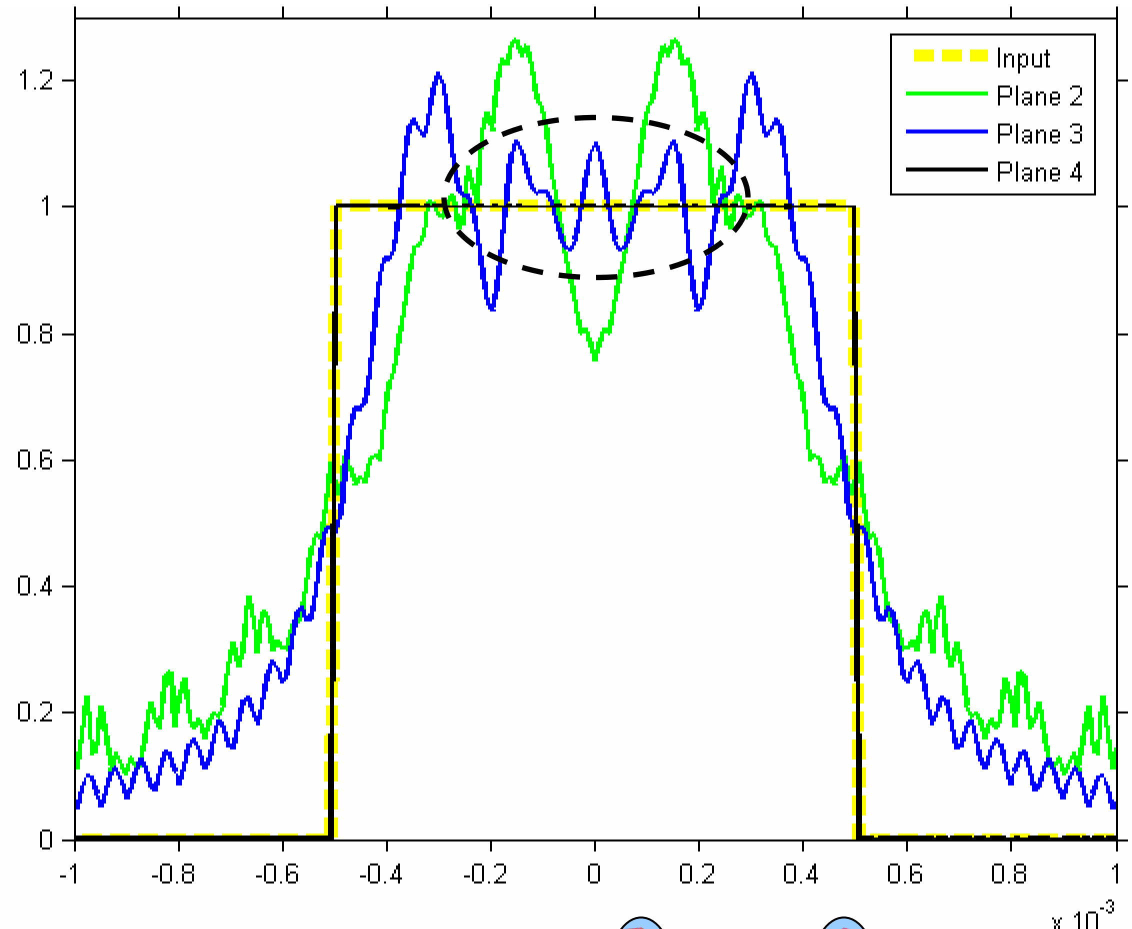
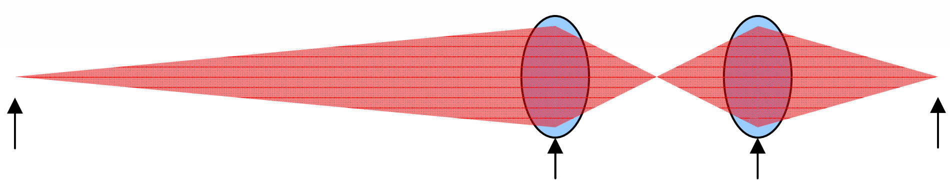
N=512, δ=9.4 µm



Minimal Sampling



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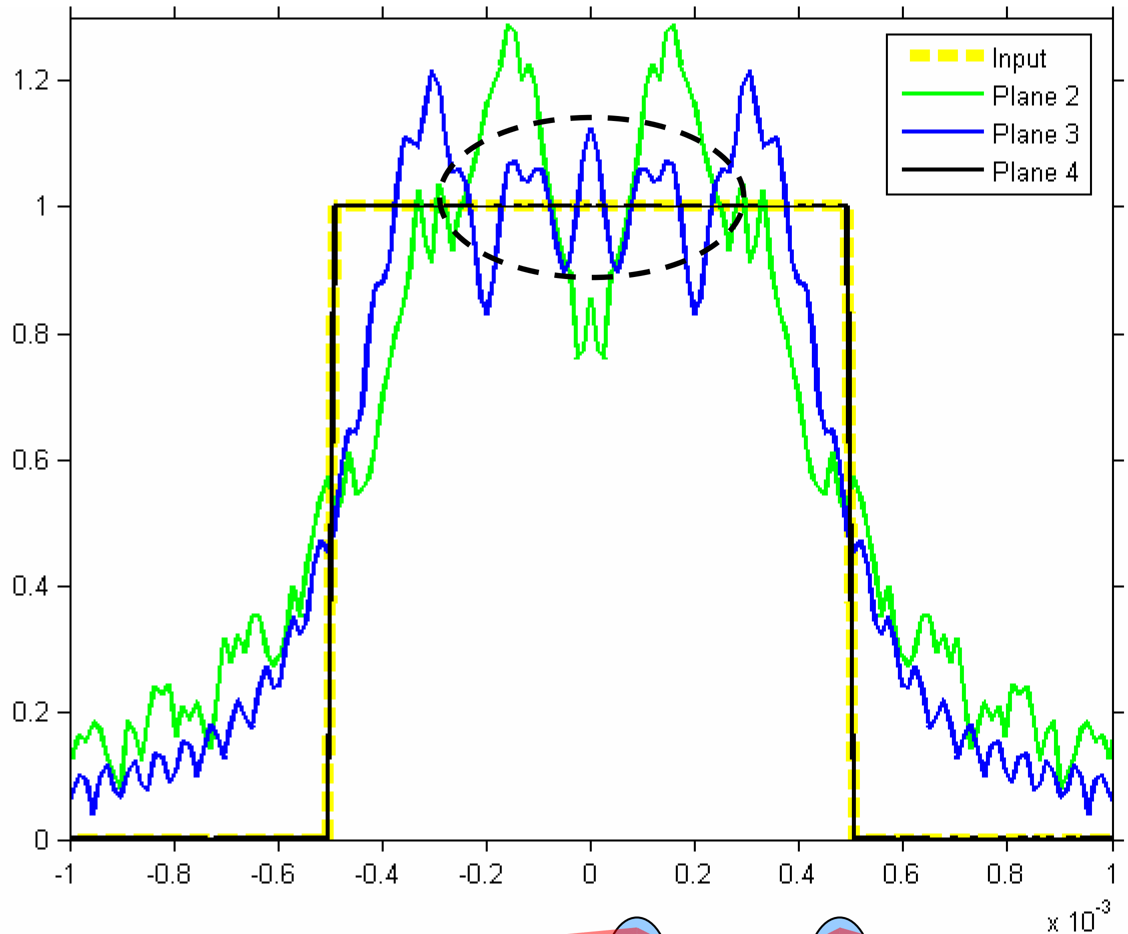
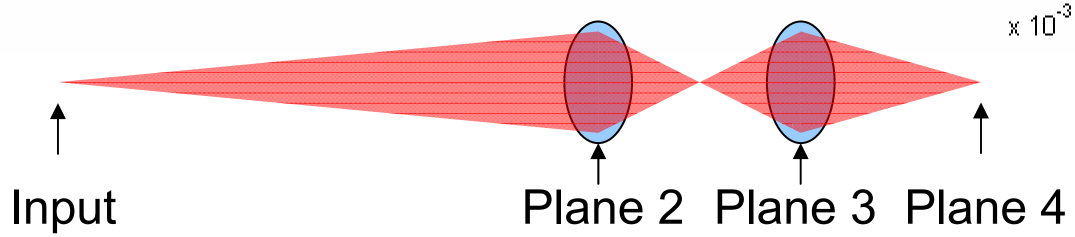
N=256, δ=13.3 μm

Under Sampled





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Conclusions

- We have devised a procedure to reduce a complex system comprised of simple optics into a pair of the most restricting apertures using the concepts of field stop and aperture stop.
- With these two apertures, a wavelength, and a distance, we can determine the mesh parameters for this system.
- Limitation: Does not include possibility of soft-edged apertures or aberrations, but they can be added.

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Questions?

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