Choosing Mesh Spacings and Mesh Dimensions for Wave Optics Simulation

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Overview

- We will present a simple step-by-step method for choosing mesh spacings and dimensions for any wave optics simulation problem. To the best of our knowledge this has never been done before.
- This method addresses both modeling correctness and computational efficiency, while leaving the user enough flexibility to deal with additional constraints.
- The method is amenable to automated implementation and well-suited for use with automated optimization techniques.
- This work has been funded in part by the Air Force Research Laboratory and the Airborne Laser Program.

Background

Fourier optics

One-step DFT propagation

<u>Two-step DFT propagation</u>

Fourier Optics

The Fresnel Diffraction Integral:

$$u_2(\vec{\rho}_2) \cong \frac{e^{ik\Delta z}}{i\lambda\Delta z} \iint d^2\vec{\rho}_1\, u_1(\vec{\rho}_1) \exp\left(\frac{ik}{2\Delta z}\left|\vec{\rho}_2 - \vec{\rho}_1\right|^2\right)$$

$$= \exp\left(i\frac{\pi}{\lambda\Delta z}\rho_2^{\,2}\right) \cdot F_{\Delta z}\left\{\exp\left(i\frac{\pi}{\lambda\Delta z}\rho_1^{\,2}\right) \cdot u_1(\vec{\rho}_1)\right\}$$

where

$$F_{\Delta z}\{u(\vec{\rho})\} = \frac{e^{ik\Delta z}}{i\lambda\Delta z} U\left(\lambda\Delta z \vec{\rho}_f\right) \quad \text{(scaled Fourier transform)}$$

$$U\left(\vec{\rho}_f\right) = F\{u(\vec{\rho})\}$$

Strictly valid only for propagation through vacuum or ideal dielectric media

Fourier Optics

The Fresnel Diffraction Integral:

$$u_2(\vec{\rho}_2) \cong \frac{e^{ik\Delta z}}{i\lambda\Delta z}\iint d^2\vec{\rho}_1\, u_1(\vec{\rho}_1)\exp\left(\frac{ik}{2\Delta z}\left|\vec{\rho}_2-\vec{\rho}_1\right|^2\right)$$

$$= \underbrace{\exp\left(i\frac{\pi}{\lambda\Delta z}\rho_2^{\,2}\right)}_{\text{quadratic phase factor}}\cdot \underbrace{F_{\Delta z}\left\{\underbrace{\exp\left(i\frac{\pi}{\lambda\Delta z}\rho_1^{\,2}\right)}_{\text{quadratic phase factor}}\cdot u_1(\vec{\rho}_1)\right\}}_{\text{scaled Fourier transform}}$$

where

$$F_{\Delta z}\{u(\vec{\rho})\} = \frac{e^{ik\Delta z}}{i\lambda\Delta z}U(\lambda\Delta z\vec{\rho}_f) \quad \text{(scaled Fourier transform)}$$

$$U(\vec{\rho}_f) = F\{u(\vec{\rho})\}$$

One-Step DFT Propagation

$$u_2(\vec{\rho}_2) \cong \exp\left(i\frac{\pi}{\lambda\Delta z}\rho_2^{\,2}\right)\cdot F_{\Delta z}\left\{\exp\left(i\frac{\pi}{\lambda\Delta z}\rho_1^{\,2}\right)\cdot u_1(\vec{\rho}_1)\right\}$$

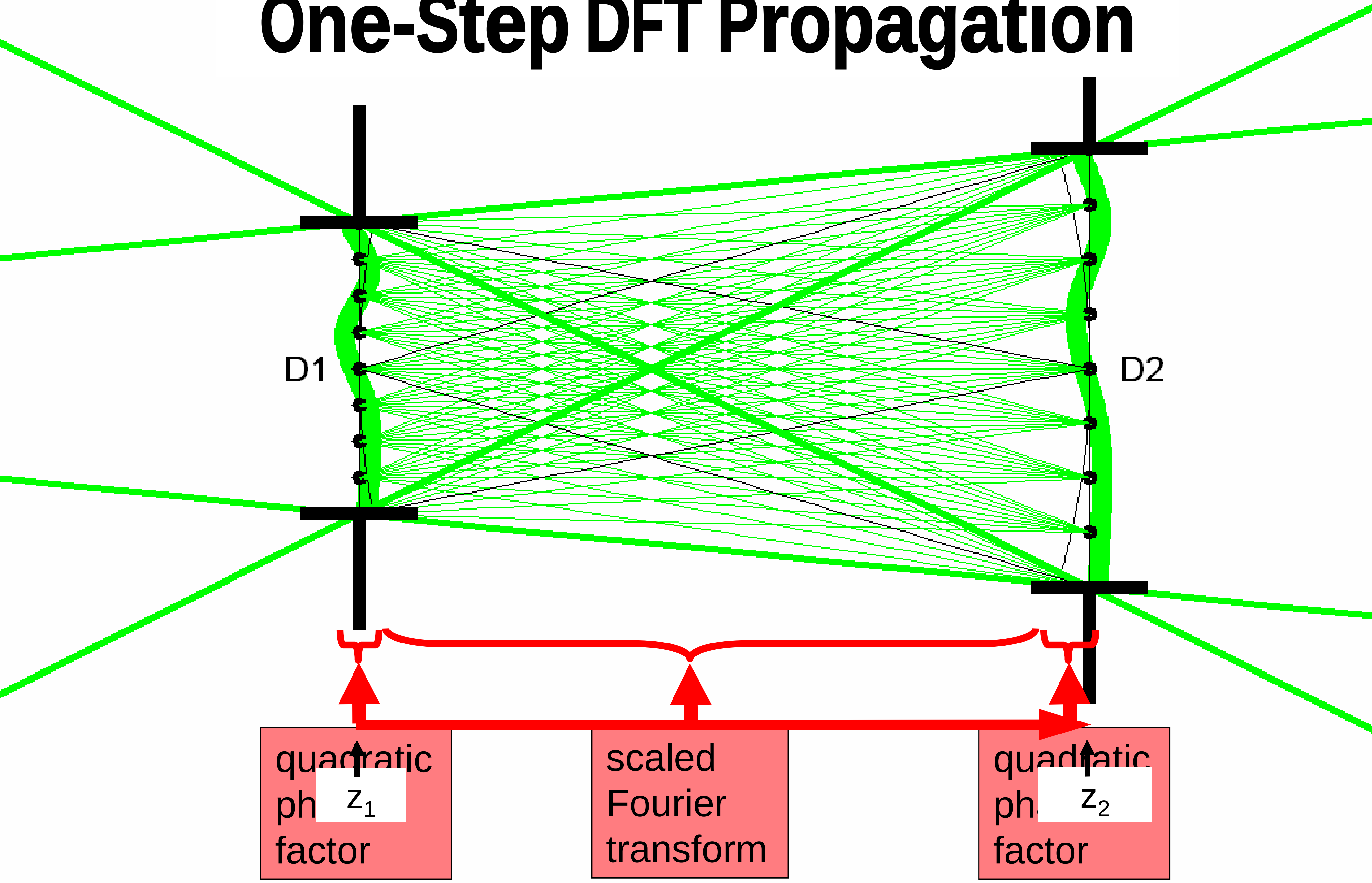
$$\Downarrow$$

$$u_{2D}(\vec{\rho}_2) \cong \exp\left(i\frac{\pi}{\lambda\Delta z}\rho_2^{\,2}\right)\cdot F_{\Delta z\,D}\left\{\exp\left(i\frac{\pi}{\lambda\Delta z}\rho_1^{\,2}\right)\cdot u_{1D}(\vec{\rho}_1)\right\}$$

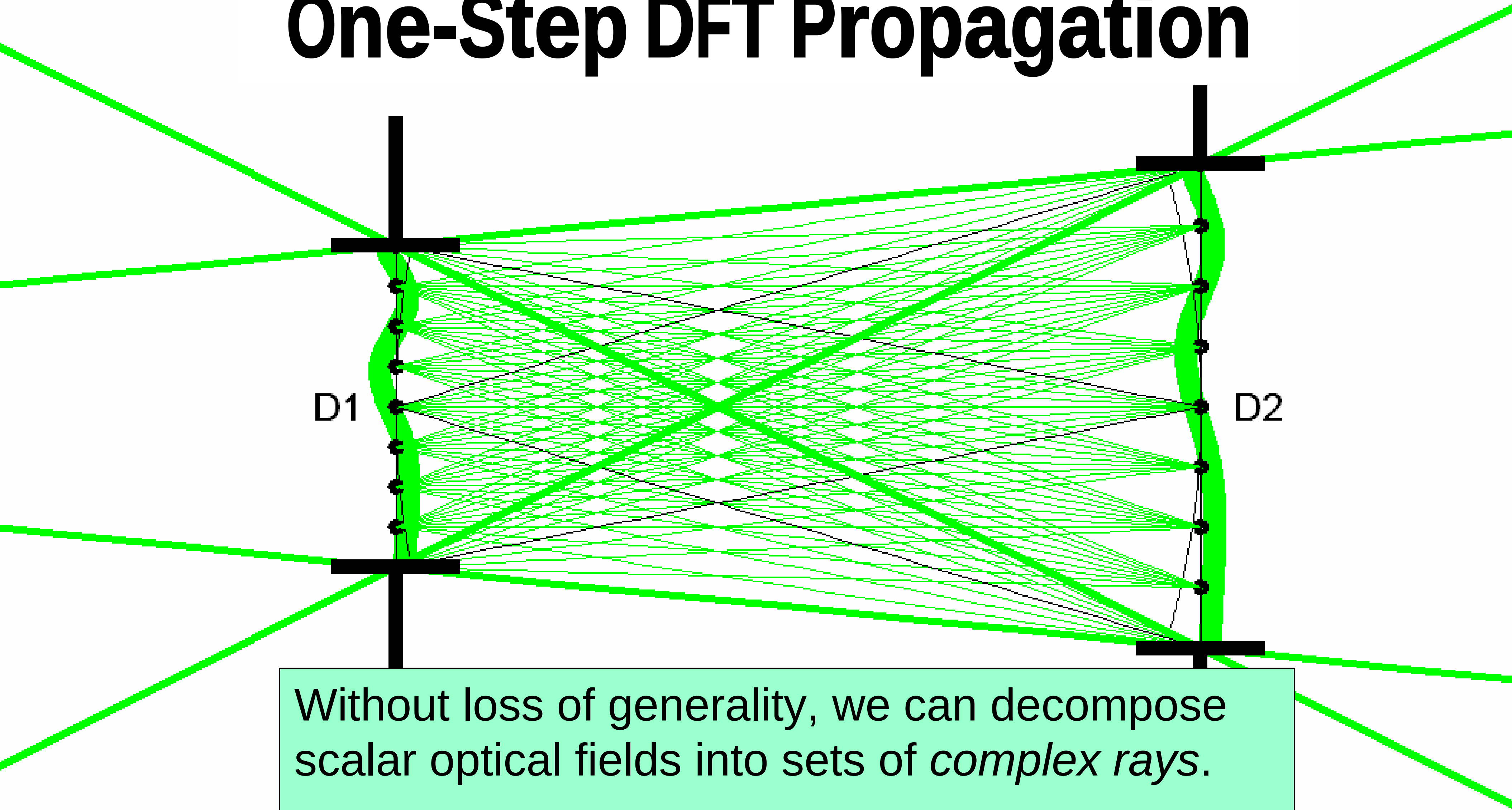
where $F_{\Delta z\,D}$ represents the Discrete Fourier Transform scaled by $\lambda\Delta z$, and u_{1D} and u_{2D} are N by N rectangular meshes with spacings δ_1 and δ_2.

$$\boxed{\delta_2 = \frac{\lambda\Delta z}{N\delta_1}}$$

One-Step DFT Propagation



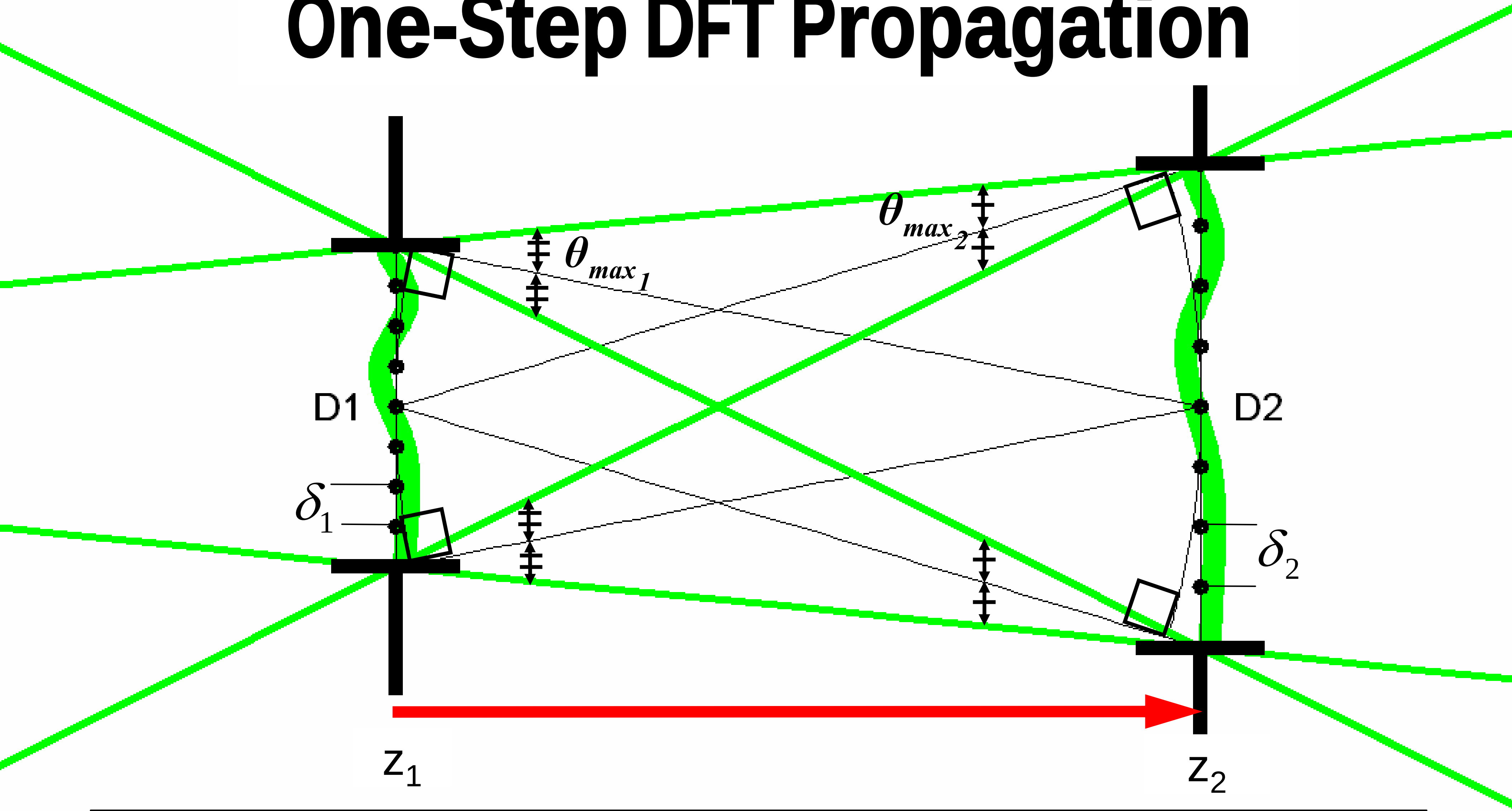
One-Step DFT Propagation



Without loss of generality, we can decompose scalar optical fields into sets of *complex rays*.

Using those rays, we can obtain constraints on the the mesh spacings and dimensions directly from the geometry of the problem.

One-Step DFT Propagation



$$\delta_1 \le \frac{\lambda}{2\theta_{\max 2}} = \frac{\lambda \Delta z}{D_2}, \qquad \delta_2 \le \frac{\lambda}{2\theta_{\max 1}} = \frac{\lambda \Delta z}{D_1}, \qquad N = \frac{\lambda \Delta z}{\delta_1 \delta_2} \ge \frac{D_1 D_2}{\lambda \Delta z}$$

One-Step DFT Propagation

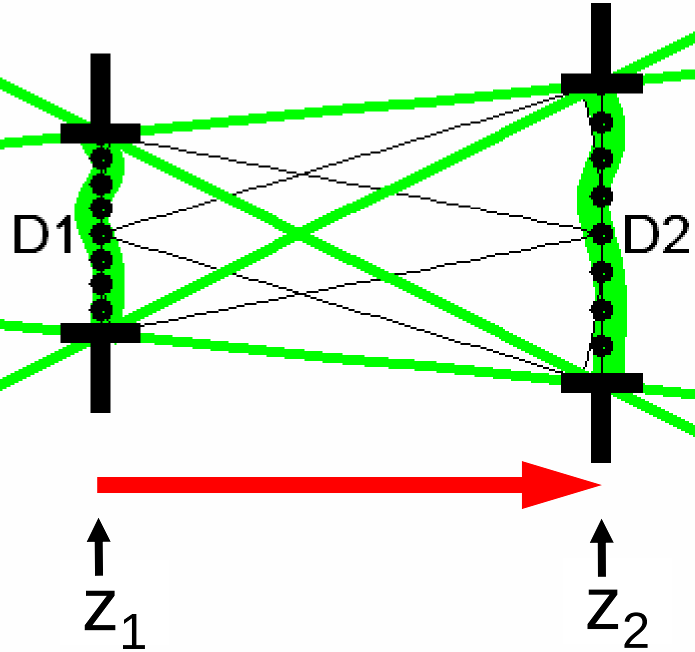
Example1:

$$\lambda = 1.0\mu\text{m},\ \Delta z = 60\text{km}$$

$$D_1 = 1.0\text{m},\ D_2 = 1.5\text{m}$$

Constraints :

$$\delta_1 \le 4.0\text{cm},\quad \delta_2 \le 6.0\text{cm},\quad N \ge 25$$



$$\delta_1 \le \frac{\lambda \Delta z}{D_2},\qquad \delta_2 \le \frac{\lambda \Delta z}{D_1},\qquad N = \frac{\lambda \Delta z}{\delta_1 \delta_2} \ge \frac{D_1 D_2}{\lambda \Delta z}$$

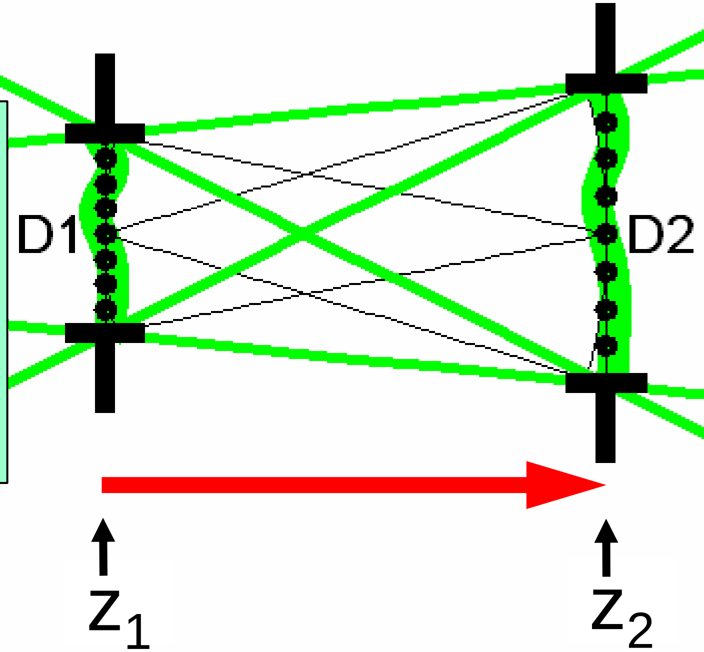
One-Step DFT Propagation

Example 2:

Same, except $D_2 = 10.0\text{m}$

Constraints:

$\delta_1 \le 6.0\text{mm},\ \ \delta_2 \le 6.0\text{cm},\ \ N \ge 167$



$$\delta_1 \le \frac{\lambda \Delta z}{D_2}, \qquad \delta_2 \le \frac{\lambda \Delta z}{D_1}, \qquad N = \frac{\lambda \Delta z}{\delta_1 \delta_2} \ge \frac{D_1 D_2}{\lambda \Delta z}$$

Two-Step DFT Propagation

$$u_{2D}(\vec{\rho}_2) \cong \exp\left(i\frac{\pi}{\lambda \Delta z}\rho_2{}^2\right) \cdot F_{\Delta z\, D}\left\{\exp\left(i\frac{\pi}{\lambda \Delta z}\rho_1{}^2\right) \cdot u_{1D}(\vec{\rho}_1)\right\}$$

$$u_{itm\,D}(\vec{\rho}_2) \cong \exp\left(i\frac{\pi}{\lambda \Delta z_1}\rho_2{}^2\right) \cdot F_{\Delta z 1 D}\left\{\exp\left(i\frac{\pi}{\lambda \Delta z_1}\rho_1{}^2\right) \cdot u_{1D}(\vec{\rho}_1)\right\}$$

$$u_{2D}(\vec{\rho}_2) \cong \exp\left(i\frac{\pi}{\lambda \Delta z_2}\rho_2{}^2\right) \cdot F_{\Delta z_2 D}\left\{\exp\left(i\frac{\pi}{\lambda \Delta z_2}\rho_1{}^2\right) \cdot u_{itm\,D}(\vec{\rho}_1)\right\}$$

where $u_{itm\,D}$ represents the optical field at some intermediate plane, z_{itm}, $\Delta z_1 = z_{itm} - z_1$, and $\Delta z_2 = z_2 - z_{itm}$.

$$\boxed{\delta_2 = \frac{\lambda \Delta z_2}{N\delta_{itm}} = \frac{\lambda \Delta z_2}{N\dfrac{\lambda \Delta z_1}{N\delta_1}} = \frac{\Delta z_2}{\Delta z_1}\delta_1 = \frac{|z_2 - z_{itm}|}{|z_1 - z_{itm}|}\delta_1}$$

Two-Step DFT Propagation

Some authors make a distinction between two different algorithms for two-step DFT propagation:

(1) Two concatenated one-step DFT propagations, as we have just described.

(2) *Frequency domain propagation*, i.e.

Perform a DFT

Multiply by a kernel

Perform an inverse DFT

However it turns out that (2) can be regarded as a special case of (1) where the two propagation steps are in opposite directions.

Two-Step DFT Propagation

For propagations between the same pair of limiting apertures two-step propagation is <u>much</u> less efficient than one-step propagation.

So why use two-step propagation?

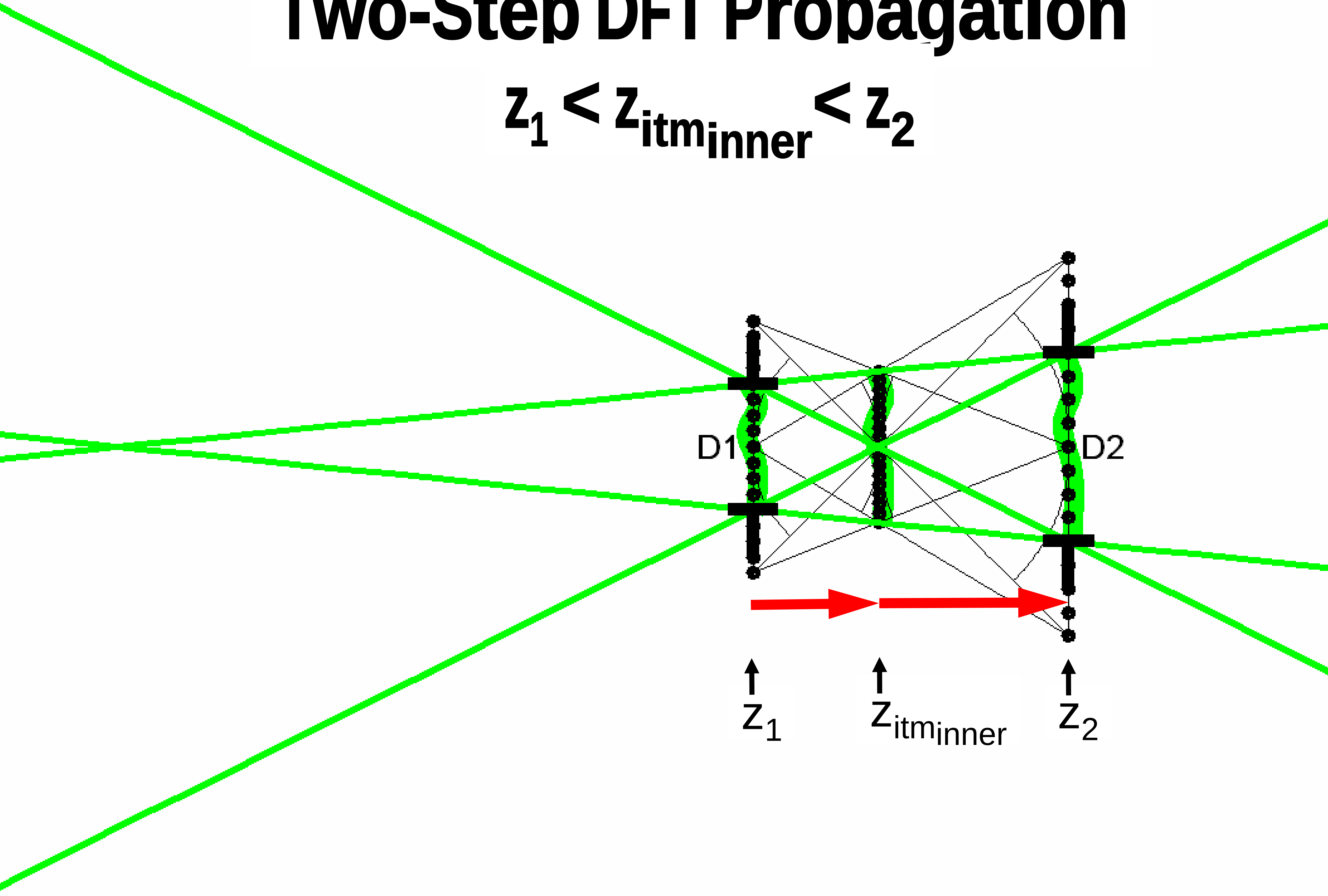
Answer:

(a) The mesh spacings at the initial and final planes can be chosen independently.

(b) It works well for propagations between any two planes along the optical path. (For one-step propagation N blows up for small Δz.)

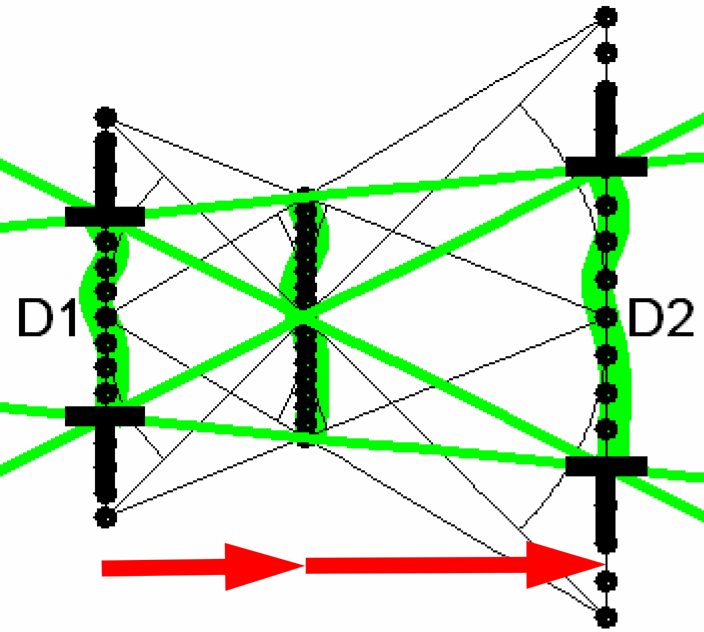
Two-Step DFT Propagation

$z_1 < z_{itm_{inner}} < z_2$



Two-Step DFT Propagation

$z_1 < z_{itm_{inner}} < z_2$



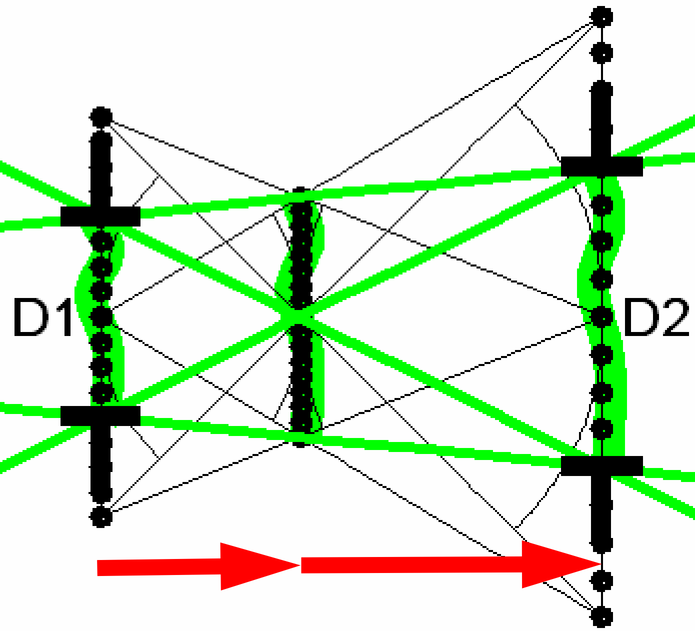
$$\delta_1 D_2 + \delta_2 D_1 \le \lambda \Delta z \quad \text{(from Nyquist)}$$

$$N \ge \frac{D_1}{\delta_1} + \frac{D_2}{\delta_2} \quad \text{(to avoid wrap-around)}$$

Two-Step DFT Propagation

$z_1 < z_{itm_{inner}} < z_2$

$$z_{itm\,inner} = z_1 + \frac{\Delta z}{1 + \frac{\delta_2}{\delta_1}}$$



$$\delta_1 D_2 + \delta_2 D_1 \le \lambda \Delta z \quad \text{(from Nyquist)}$$

$$N \ge \frac{D_1}{\delta_1} + \frac{D_2}{\delta_2} \quad \text{(to avoid wrap-around)}$$

Two-Step DFT Propagation

$z_1 < z_{itm_{inner}} < z_2$

To minimize N:

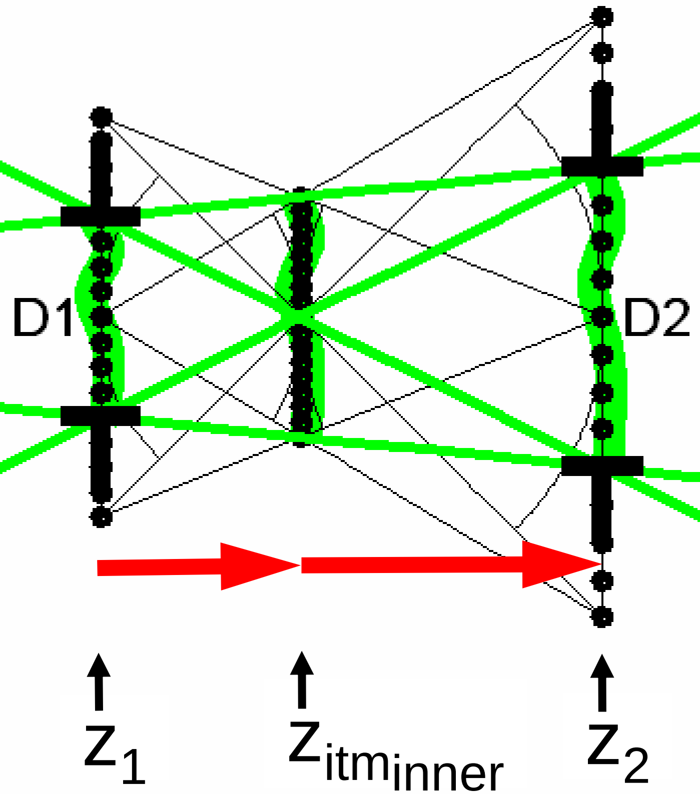
$$\delta_1 = \frac{\lambda \Delta z}{2D_2}, \quad \delta_2 = \frac{\lambda \Delta z}{2D_1}$$

$$N \geq \frac{4D_1 D_2}{\lambda \Delta z}$$

To make $\delta_1 = \delta_2 = \delta$:

$$\delta_1 = \delta_2 = \delta \leq \frac{\lambda \Delta z}{D_1 + D_2}$$

$$N \geq \frac{D_1 + D_2}{\delta}$$



$$\delta_1 D_2 + \delta_2 D_1 \leq \lambda \Delta z, \qquad N \geq \frac{D_1}{\delta_1} + \frac{D_2}{\delta_2}$$

Two-Step DFT Propagation

$z_1 < z_{itm_{inner}} < z_2$

Example1:

$\lambda = 1.0\mu\text{m}, \ \Delta z = 60\text{km}$

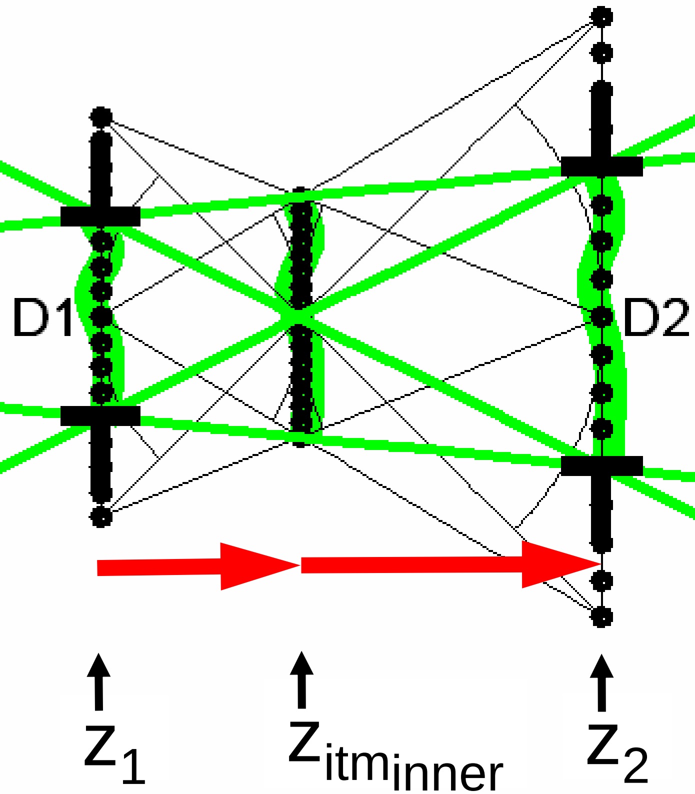
$D_1 = 1.0\text{m}, \quad D_2 = 1.5\text{m}$

Minimizing N:

$\delta_1 = 2.0\text{cm}, \quad \delta_2 = 3.0\text{cm}, \quad N = 100$

Making $\delta_1 = \delta_2$:

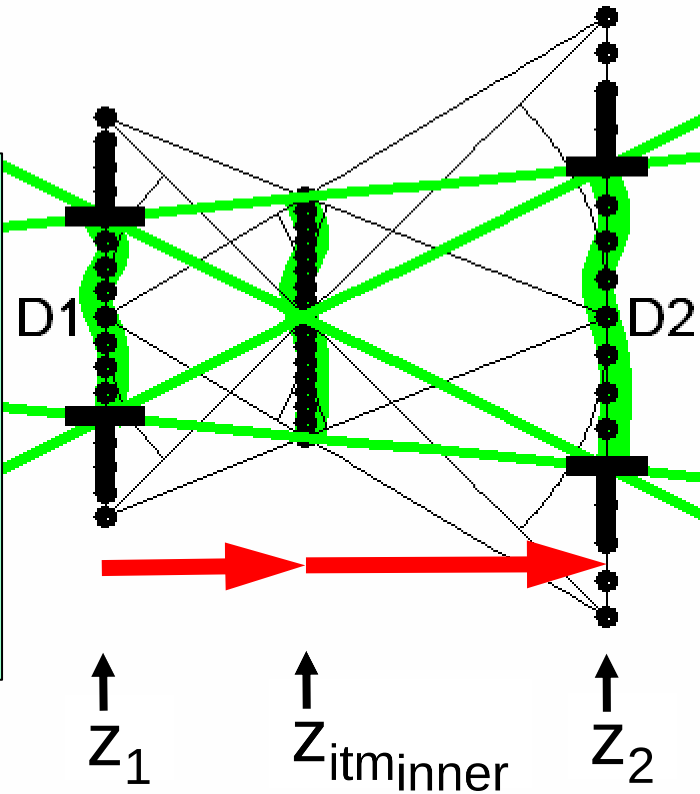
$\delta_1 = 2.4\text{cm}, \quad \delta_2 = 2.4\text{cm}, \quad N \geq 105$



$$\delta_1 D_2 + \delta_2 D_1 \leq \lambda \Delta z, \qquad N \geq \frac{D_1}{\delta_1} + \frac{D_2}{\delta_2}$$

Two-Step DFT Propagation

$z_1 < z_{itm_{inner}} < z_2$



Example 2 :

Same, except $D_2 = 10.0\text{m}$

Minimizing N :

$\delta_1 = 3.0\text{mm},\ \ \delta_2 = 3.0\text{cm},\ \ N = 667$

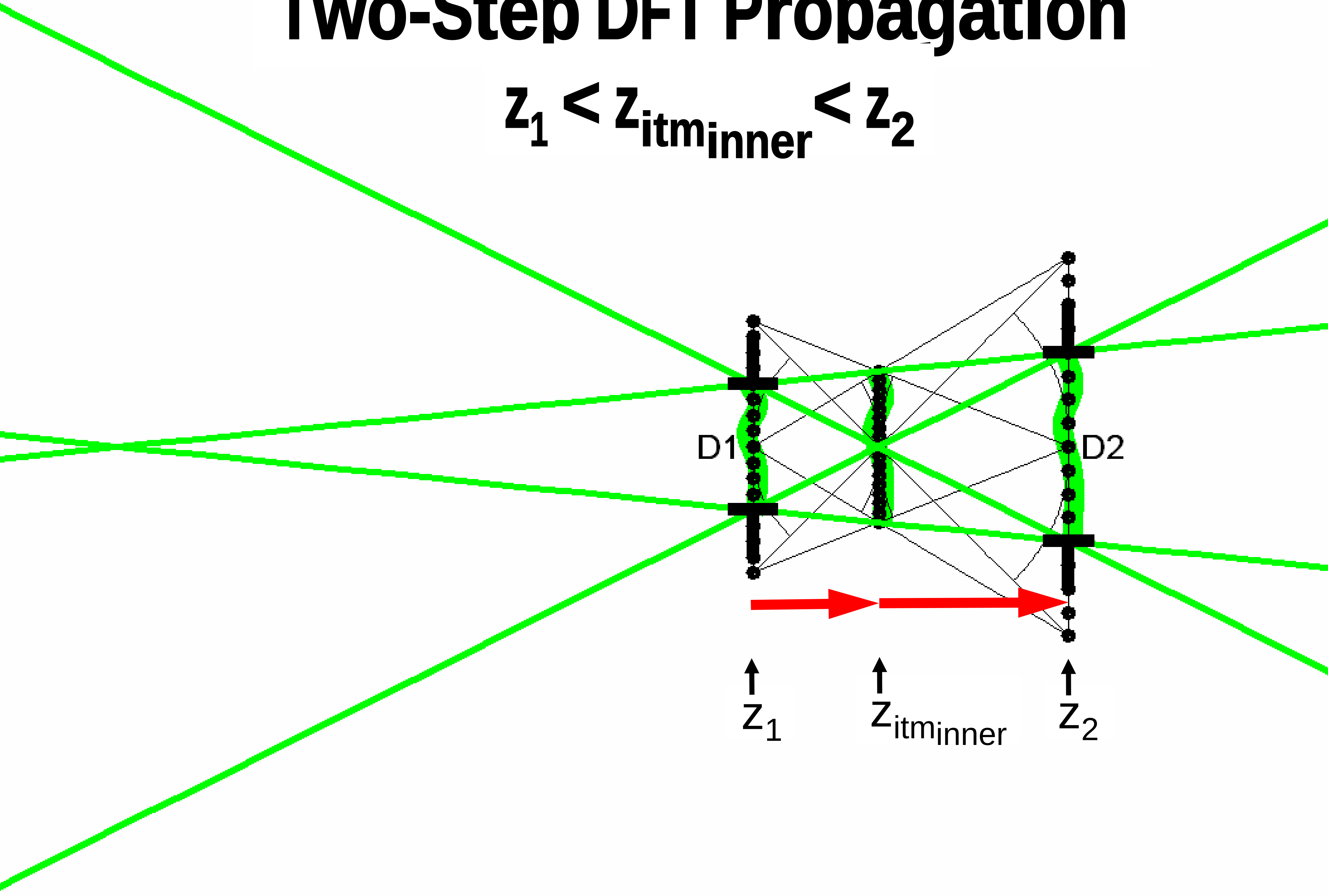
Making $\delta_1 = \delta_2$:

$\delta_1 = 5.5\text{mm},\ \ \delta_2 = 5.5\text{mm},\ N \geq 2017$

$$\delta_1 D_2 + \delta_2 D_1 \leq \lambda \Delta z, \qquad N \geq \frac{D_1}{\delta_1} + \frac{D_2}{\delta_2}$$

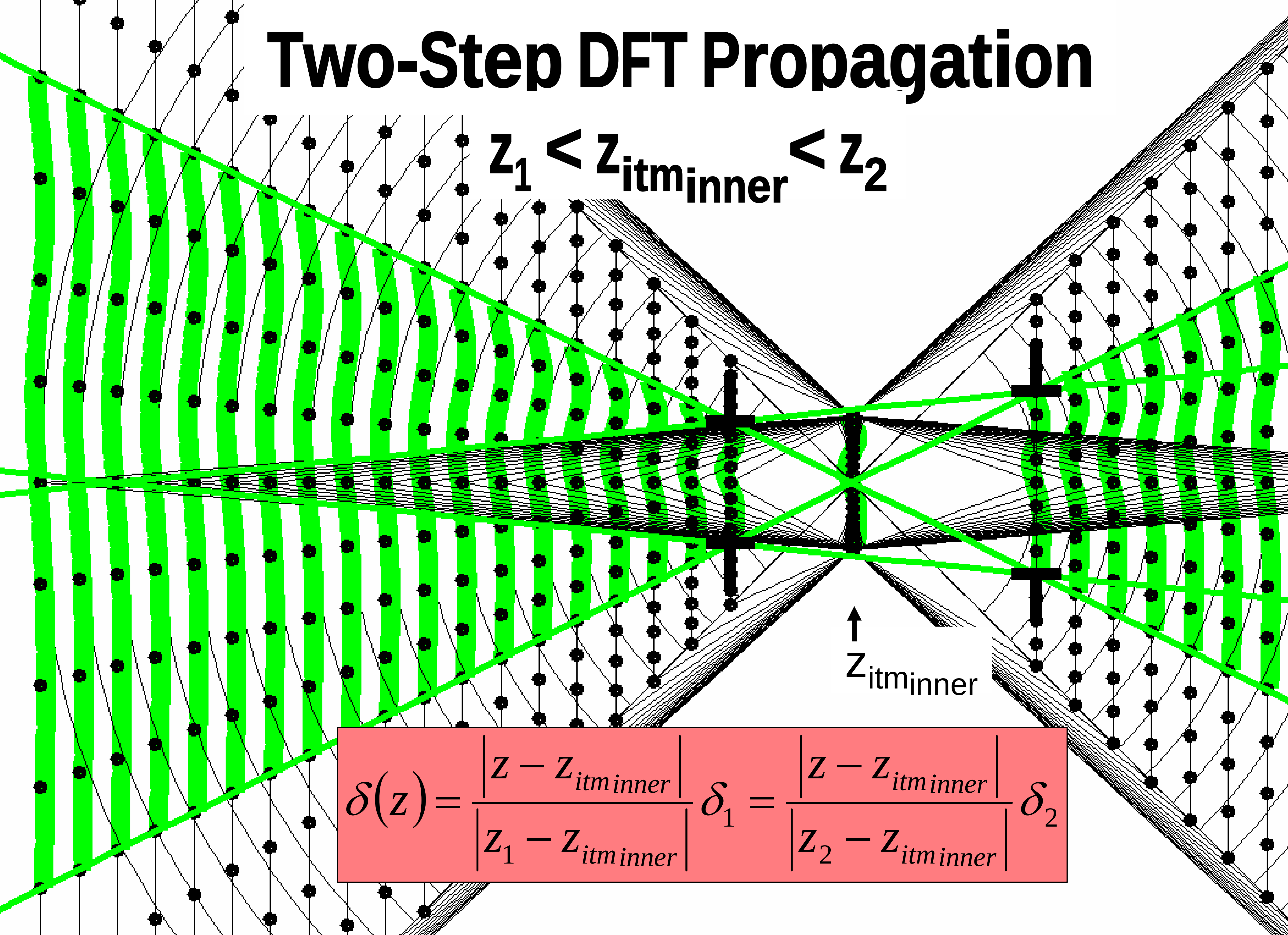
Two-Step DFT Propagation

$z_1 < z_{itm_{inner}} < z_2$



Two-Step DFT Propagation

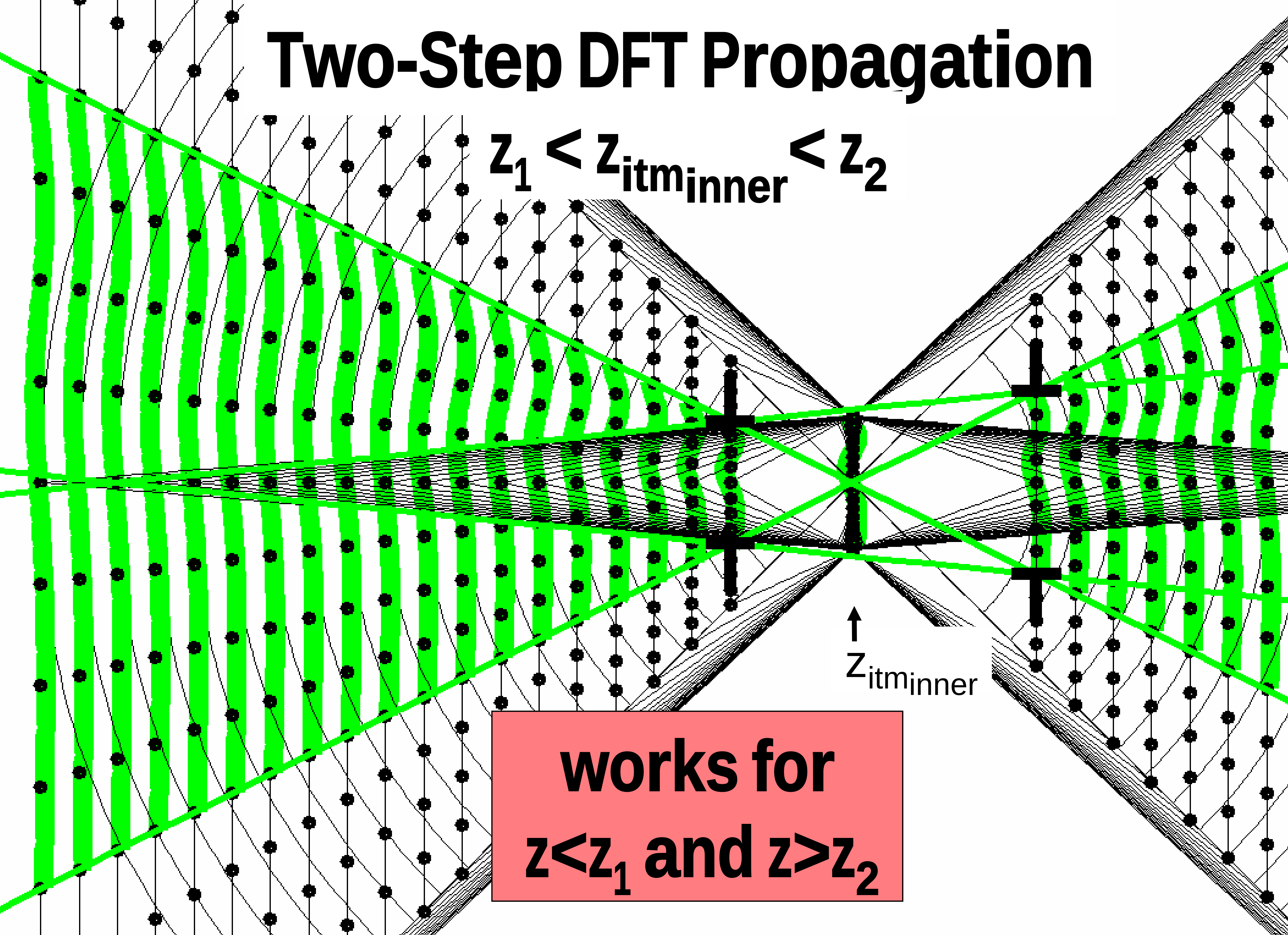
$$z_1 < z_{itm_{inner}} < z_2$$



$$\delta(z) = \frac{\left|z - z_{itm\,inner}\right|}{\left|z_1 - z_{itm\,inner}\right|}\delta_1 = \frac{\left|z - z_{itm\,inner}\right|}{\left|z_2 - z_{itm\,inner}\right|}\delta_2$$

Two-Step DFT Propagation

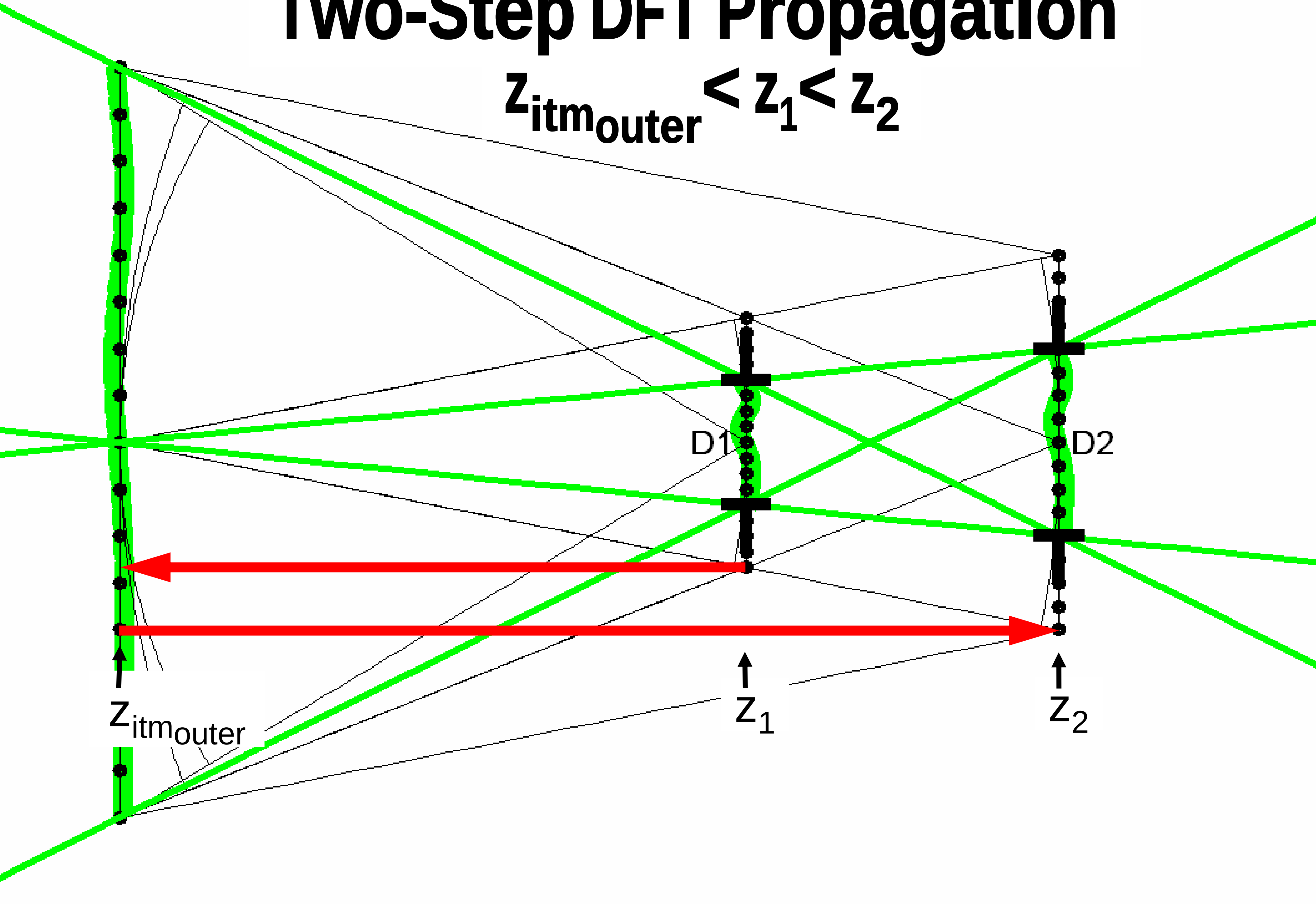
$z_1 < z_{itm_{inner}} < z_2$



works for $z<z_1$ and $z>z_2$

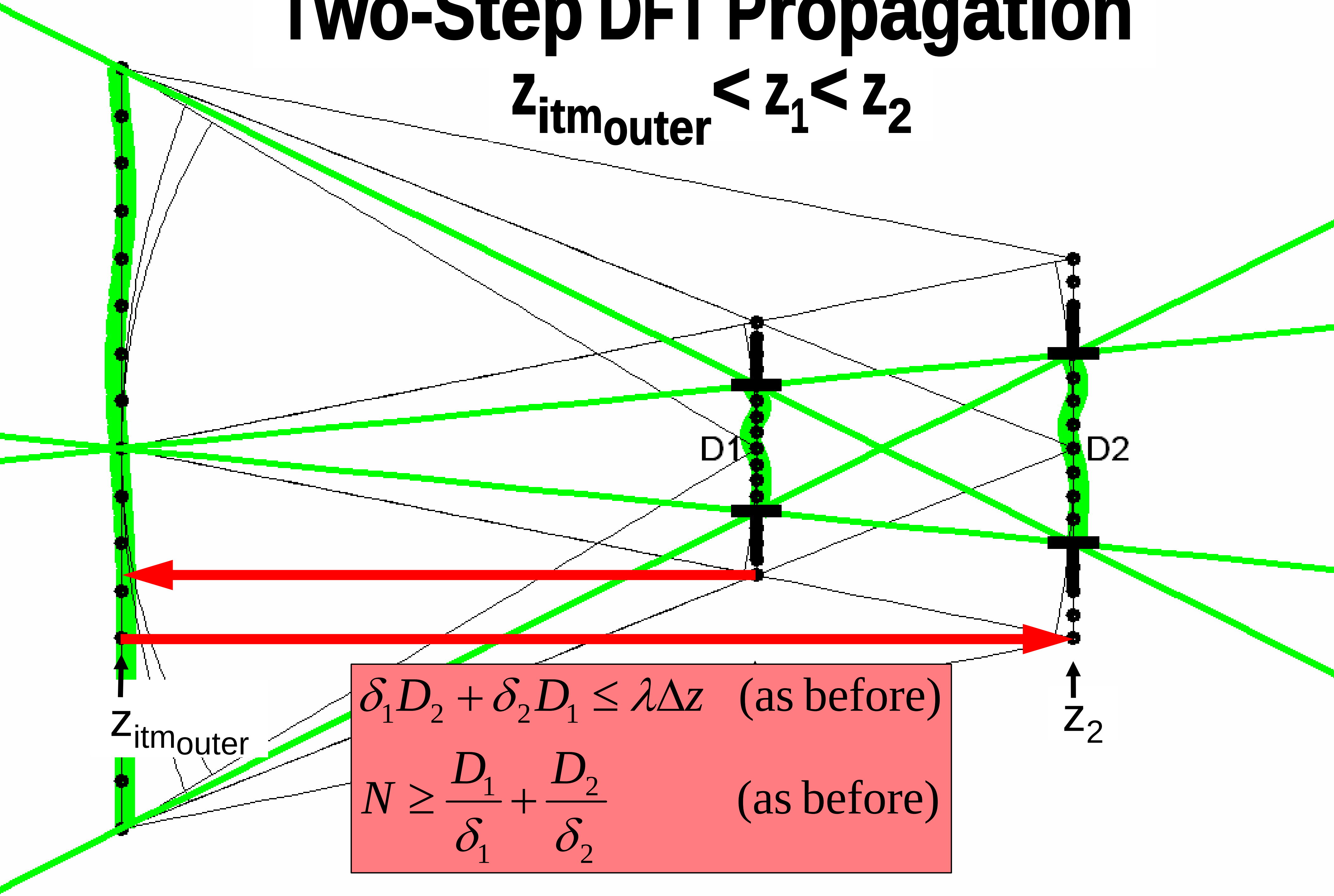
Two-Step DFT Propagation

$z_{itm_{outer}} < z_1 < z_2$



Two-Step DFT Propagation

$z_{itm_{outer}} < z_1 < z_2$

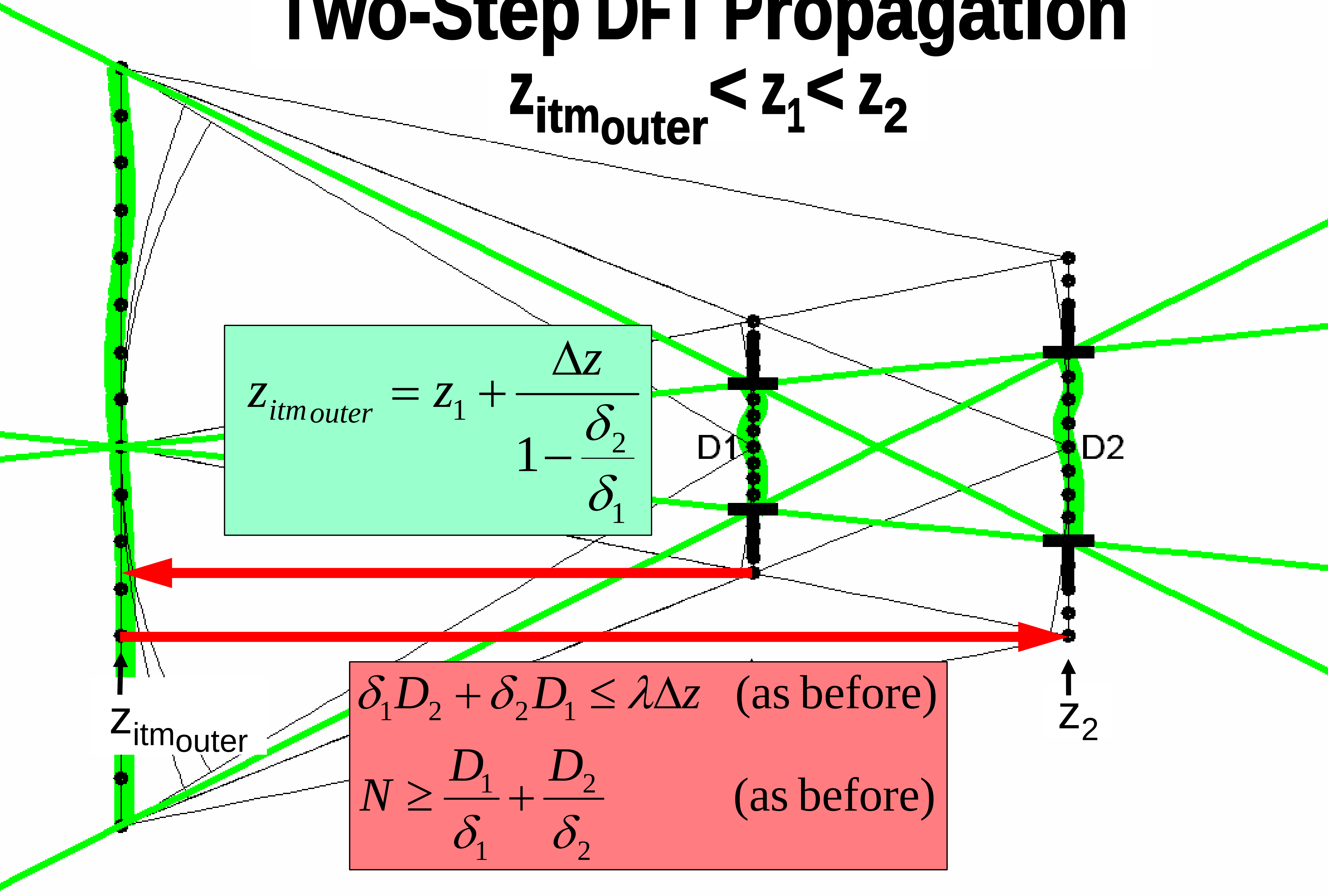


$$\delta_1 D_2 + \delta_2 D_1 \le \lambda \Delta z \quad \text{(as before)}$$

$$N \ge \frac{D_1}{\delta_1} + \frac{D_2}{\delta_2} \quad \text{(as before)}$$

Two-Step DFT Propagation

$z_{itm_{outer}} < z_1 < z_2$



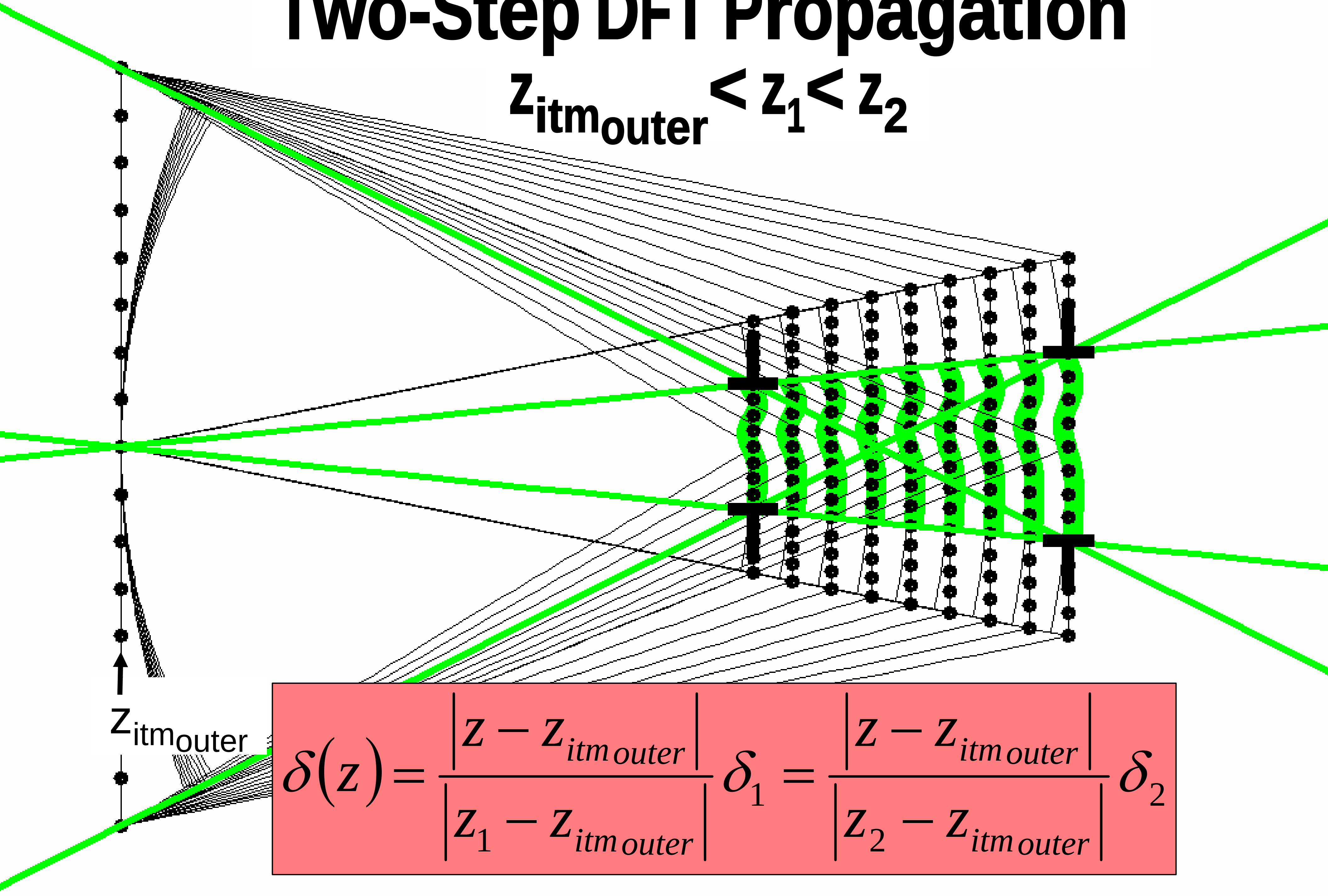
$$z_{itm\,outer} = z_1 + \frac{\Delta z}{1 - \dfrac{\delta_2}{\delta_1}}$$

$$\delta_1 D_2 + \delta_2 D_1 \le \lambda \Delta z \quad \text{(as before)}$$

$$N \ge \frac{D_1}{\delta_1} + \frac{D_2}{\delta_2} \quad \text{(as before)}$$

Two-Step DFT Propagation

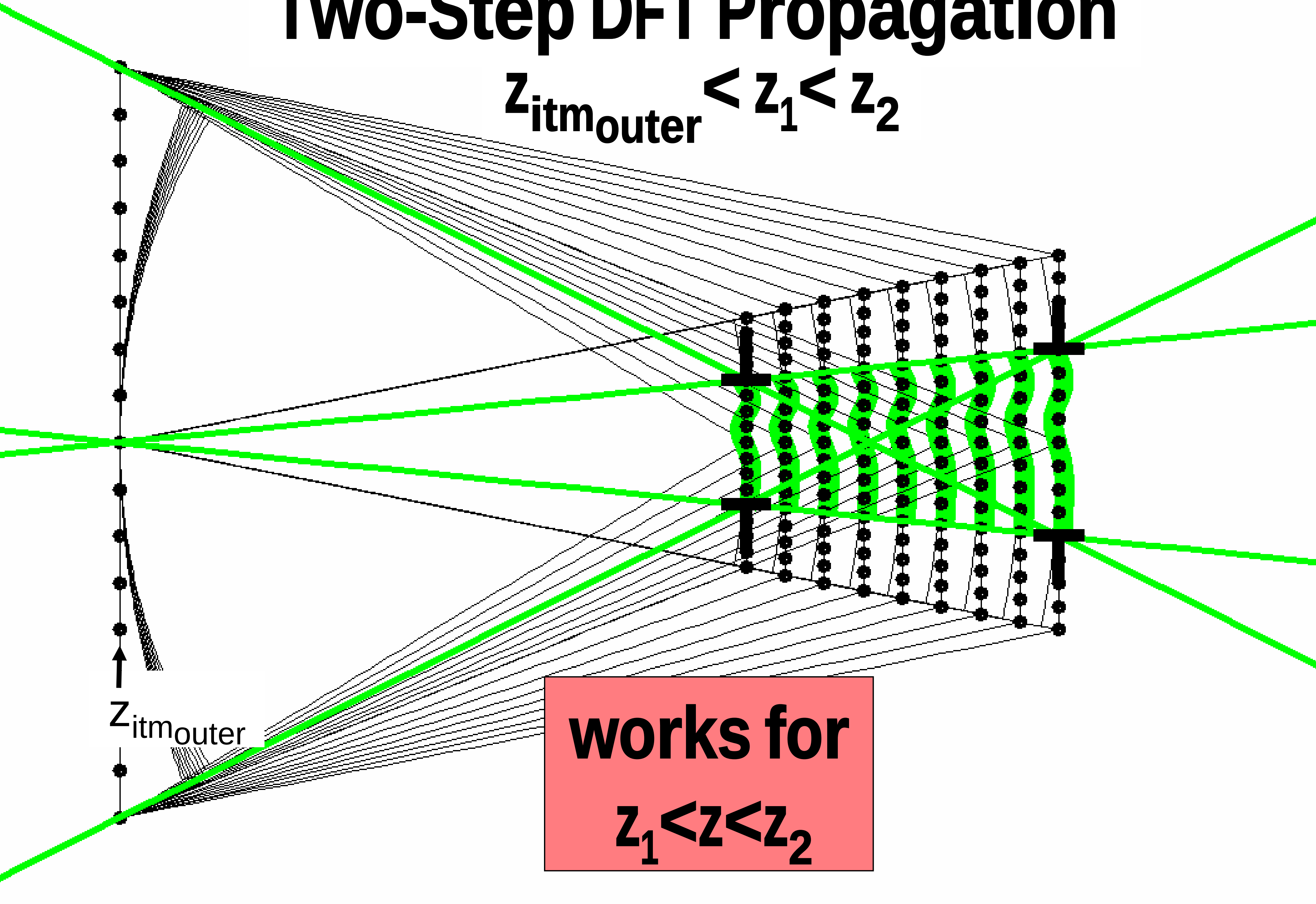
$$z_{itm_{outer}} < z_1 < z_2$$



$$\delta(z) = \frac{\left|z - z_{itm\,outer}\right|}{\left|z_1 - z_{itm\,outer}\right|}\delta_1 = \frac{\left|z - z_{itm\,outer}\right|}{\left|z_2 - z_{itm\,outer}\right|}\delta_2$$

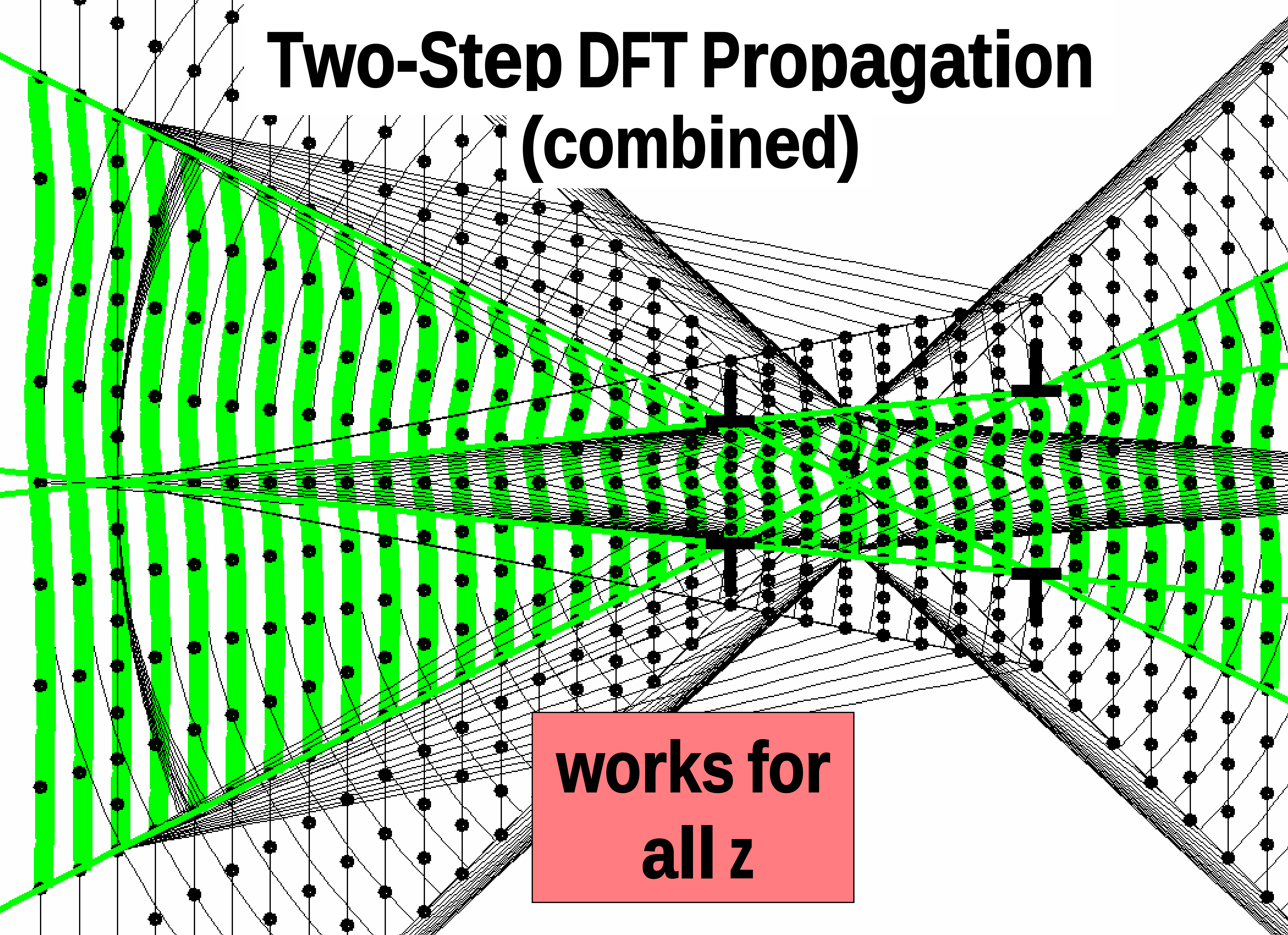
Two-Step DFT Propagation

$z_{itm_{outer}} < z_1 < z_2$



works for $z_1 < z < z_2$

Two-Step DFT Propagation (combined)



works for all z

Two-Step DFT Propagation

Bottom line: once we have identified two limiting apertures we can construct a single consistent geometry that works for propagations between any two planes, using two different intermediate planes, one for $z \in [z_1, z_2]$, one for $z \notin [z_1, z_2]$.

δ_1, δ_2, and N must be chosen to satisfy the following:

$$\delta_1 D_2 + \delta_2 D_1 \le \lambda \Delta z, \qquad N \ge \frac{D_1}{\delta_1} + \frac{D_2}{\delta_2}$$

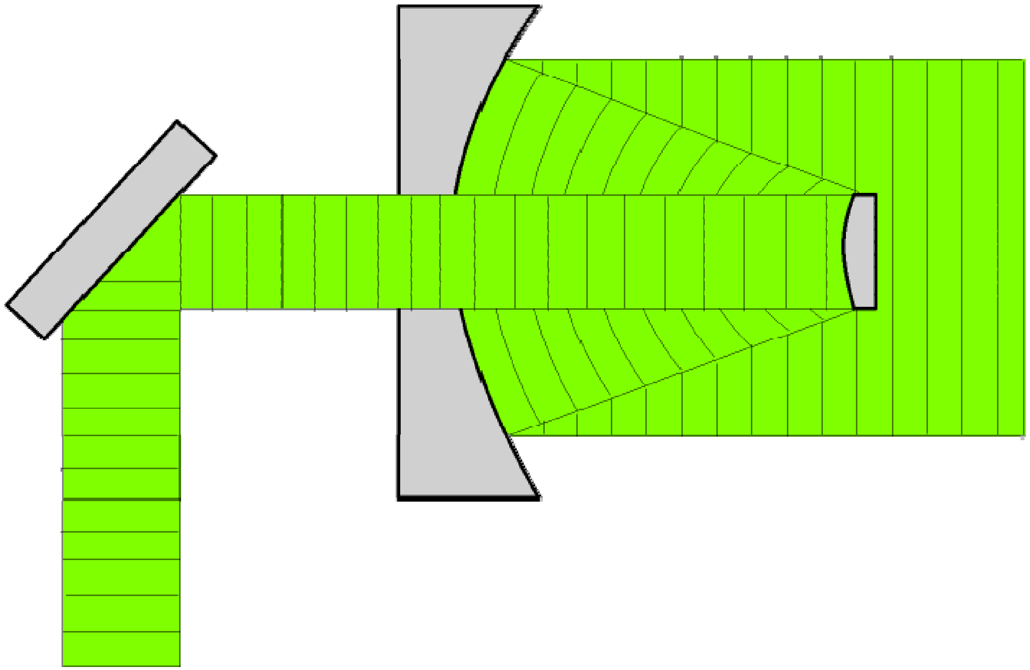
This result is strictly valid only for propagation through vacuum or ideal dielectric media.

A General Method for Choosing Mesh Spacings and Mesh Dimensions

We now have a method for choosing mesh spacings and dimensions for the special case of propagation through vacuum or ideal dielectric media, given two limiting apertures.

Next, we will present a simple step-by-step procedure to reduce any wave optics simulation problem, including propagation through optical systems and aberrating media, to one or more instances of the special case.

Step 1. Remove any lenses and mirrors

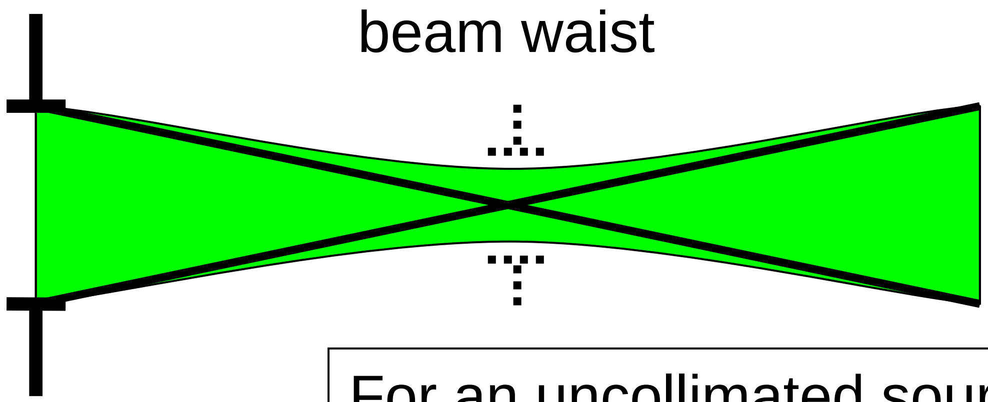


To first order, ordinary lenses and mirrors operate only on the overall tilt and/or curvature of wavefronts passing through the optical system.

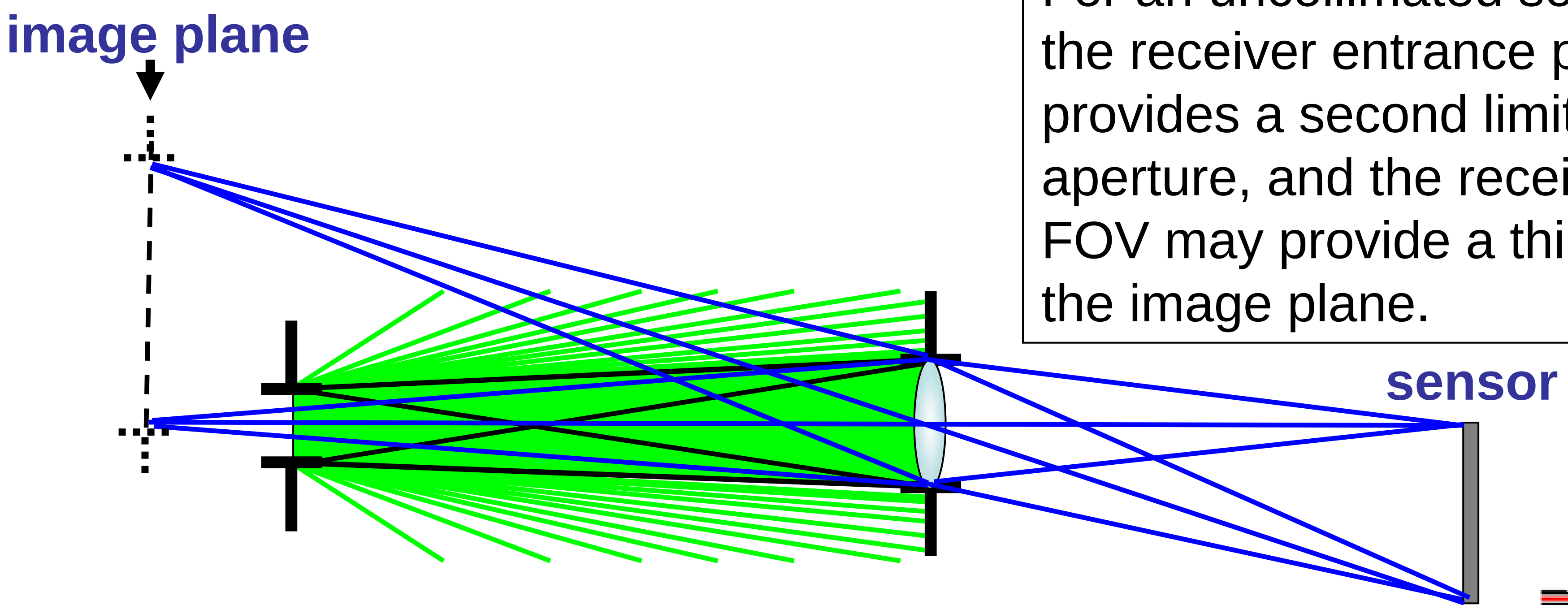
For our purposes these effects can be removed picking some one plane to start from, e.g. the source plane, and then replacing all apertures and aberrating effects with their images, as seen through the intervening lenses and mirrors.

Step 2. Identify two or more limiting apertures from a priori information.

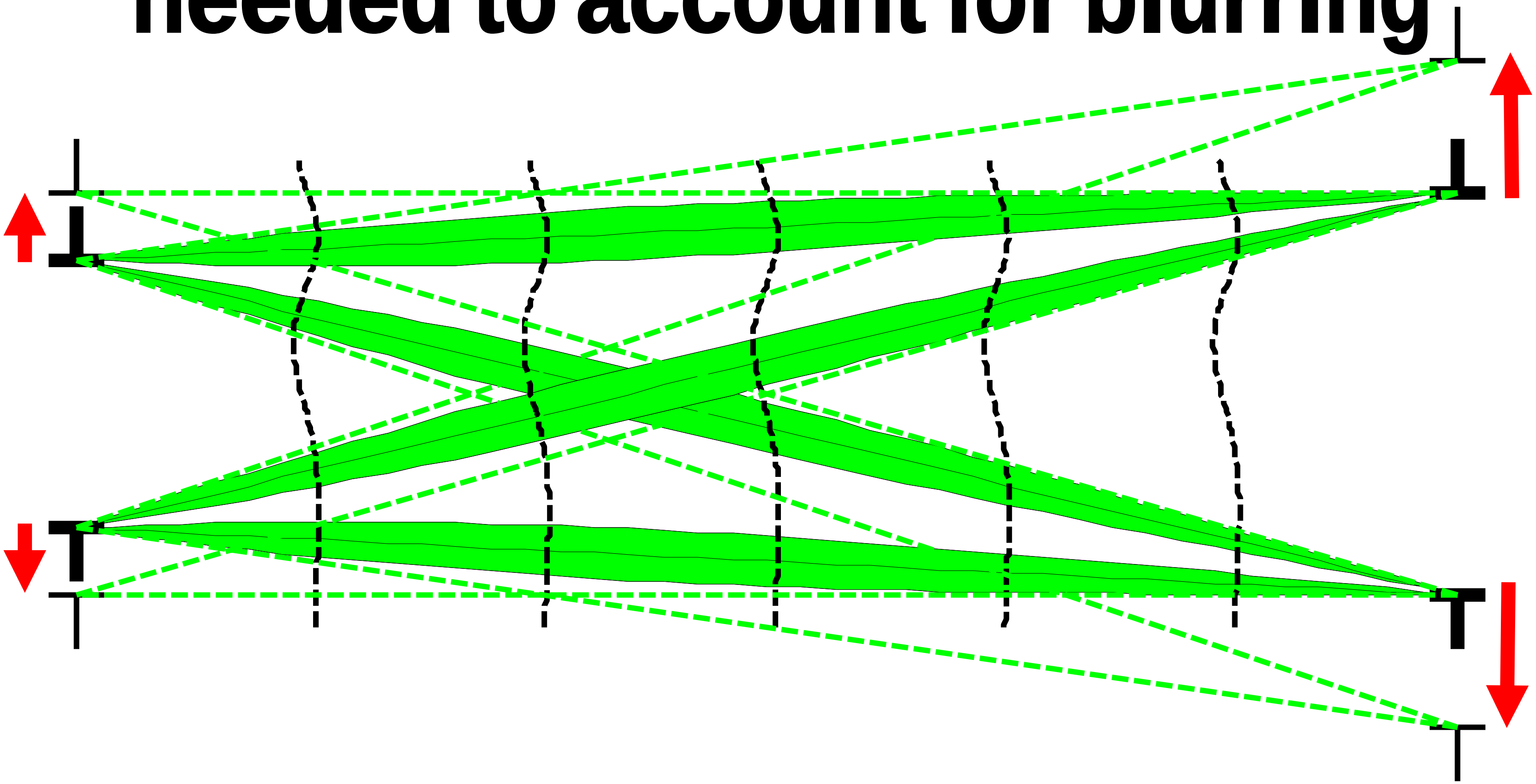
A collimated source can be thought of as having a second limiting aperture at or near the beam waist.



For an uncollimated source, the receiver entrance pupil provides a second limiting aperture, and the receiver FOV may provide a third, at the image plane.

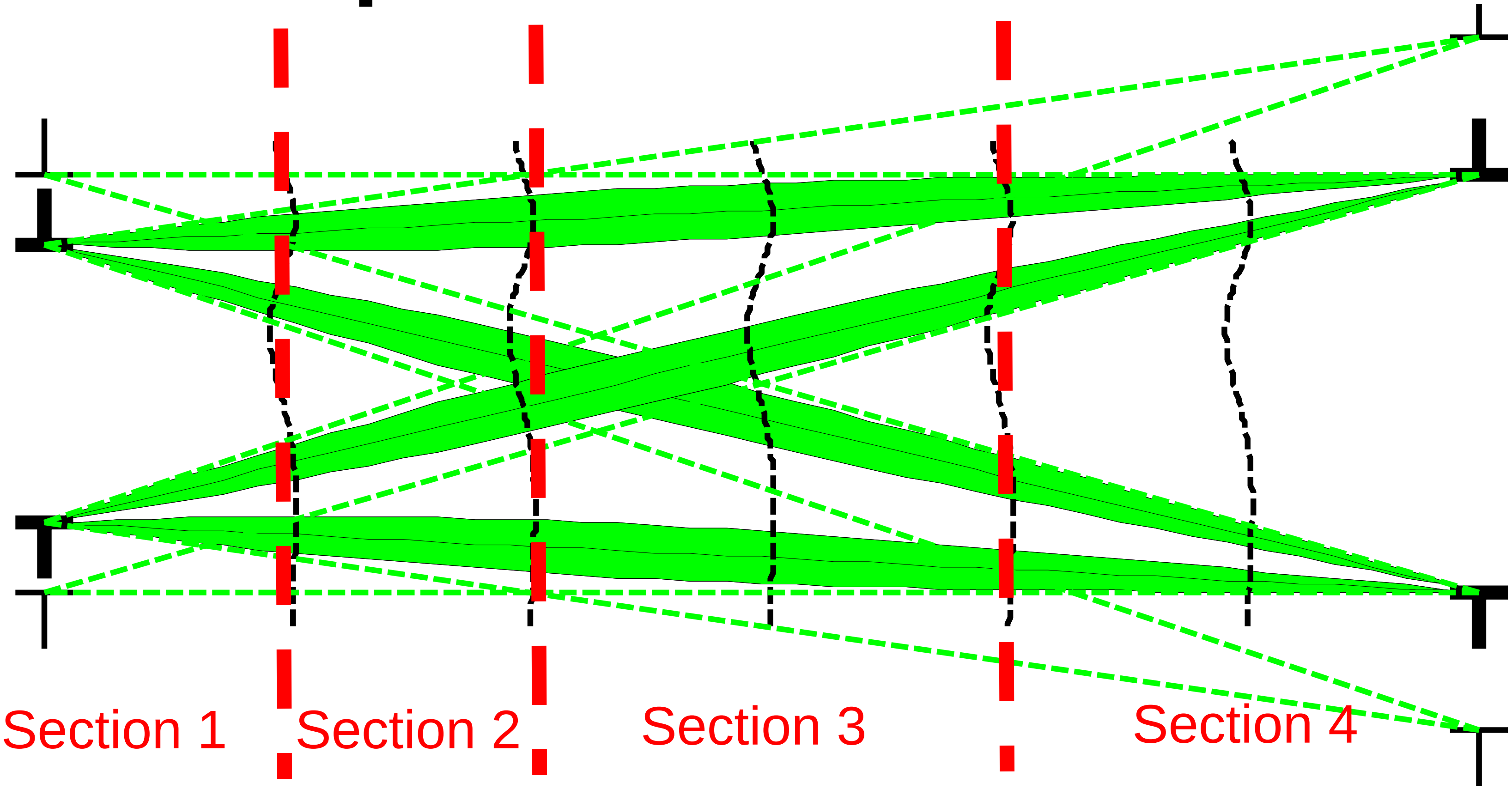


Step 3. Enlarge the apertures as needed to account for blurring



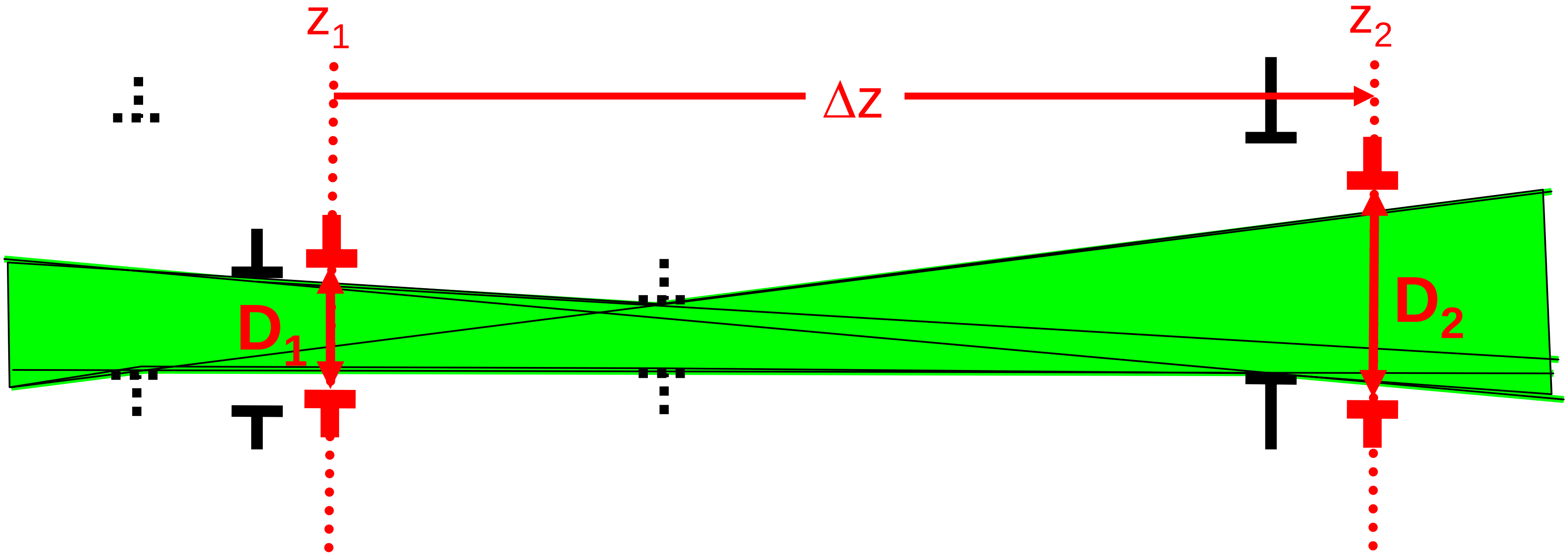
Blurring effects due to diffraction or propagation through aberrating media have the effect of enlarging the apparent size of the source aperture, as seen from the receiver, and vice versa.

Step 3a. In some cases, it may be useful to break the path into two or more sections.



Blurring effects vary with position, changing the sizes of the blurred apertures. For example, at the source the set of rays to be modeled is limited by the <u>unblurred</u> source aperture and the <u>blurred</u> receiver aperture, while at the receiver it is limited by the <u>unblurred</u> receiver aperture and the blurred <u>source</u> aperture.

Step 4. Select exactly two apertures to use in choosing spacings and dimensions



These two apertures can be the same as two of the limiting apertures identified earlier, but they need not be; instead they could be placed at different planes, chosen for convenience.

They should be chosen such that they both capture all light of interest and, to keep *N* reasonable, little light not of interest.

Step 5. Choose the mesh spacings and dimensions to satisfy the following:

$$\delta_1 D_2 + \delta_2 D_1 \le \lambda \Delta z \quad \text{(from Nyquist)}$$

$$N \ge \frac{D_1}{\delta_1} + \frac{D_2}{\delta_2} \qquad \text{(to avoid wrap - around)}$$

To minimize N, choose as follows:

$$\delta_1 = \frac{\lambda \Delta z}{2D_2}, \quad \delta_2 = \frac{\lambda \Delta z}{2D_1}, \quad N \ge \frac{4D_1 D_2}{\lambda \Delta z}$$

To make $\delta_1 = \delta_2$, choose as follows:

$$\delta_1 = \delta_2 = \delta \le \frac{\lambda \Delta z}{D_1 + D_2}, \quad N \ge \frac{D_1 + D_2}{\delta}$$

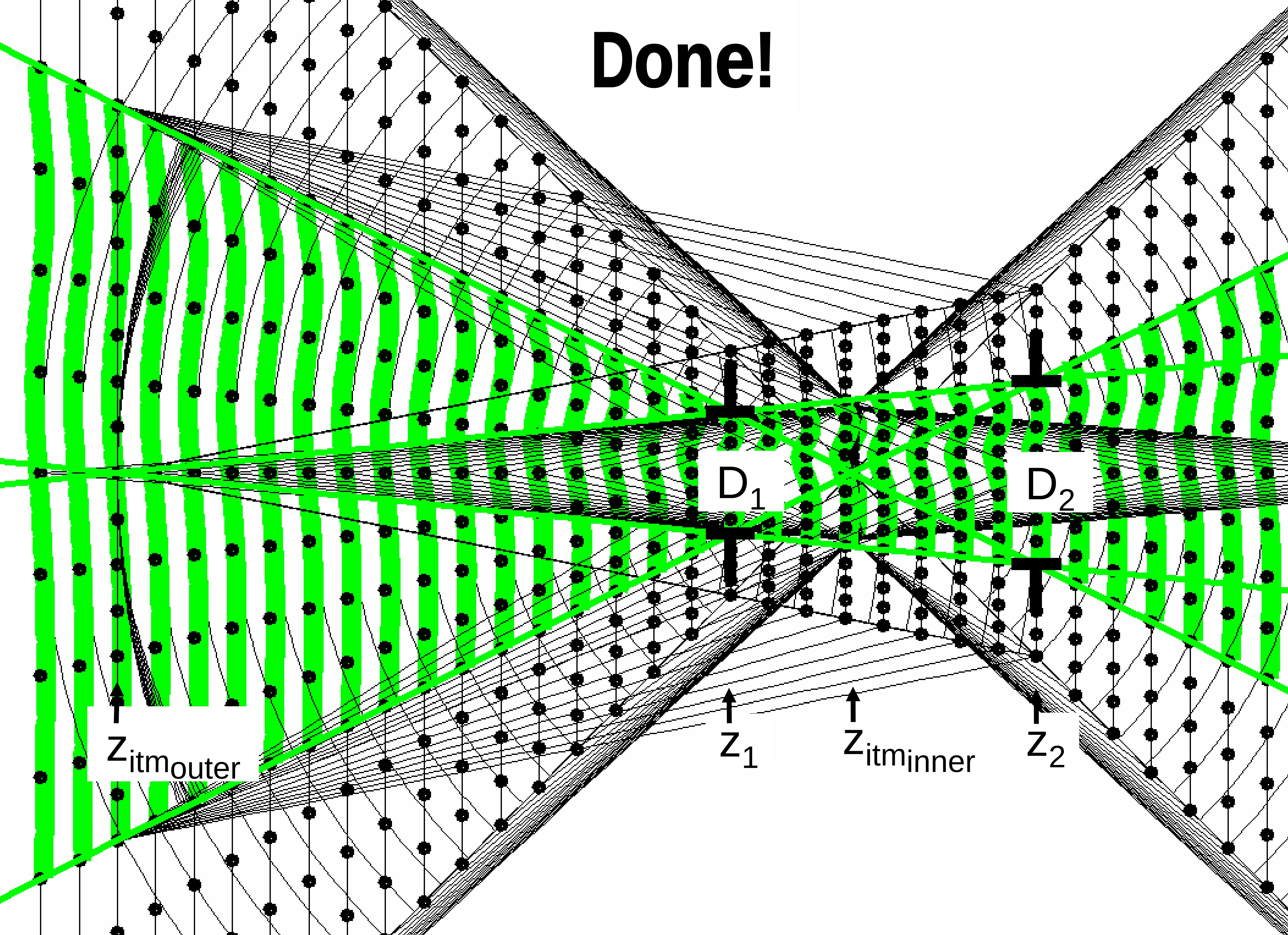
Step 6. Compute the locations of two *intermediate planes* to be used in two-step DFT propagations:

$$z_{itm_{inner}} = z_1 + \frac{\Delta z}{1 + \dfrac{\delta_2}{\delta_1}} \qquad z_{itm_{outer}} = z_1 + \frac{\Delta z}{1 - \dfrac{\delta_2}{\delta_1}}$$

The *inner* intermediate plane lies *inside* the two aperture planes and is used for propagations *outside* those planes.

The *outer* intermediate plane lies *outside* the two aperture planes and is used for propagations *inside* those planes

Done!



Summary and Conclusions

- **We have presented a simple step-by-step method for choosing mesh spacings and dimensions for wave optics simulation.**
- **This method addresses both modeling correctness and computational efficiency, while leaving the user enough flexibility to deal with additional constraints.**
- **The method is amenable to automated implementation and well-suited for use with automated optimization techniques.**
- **Caveat: there are other important issues that must be taken into account in order to obtain correct results using wave optics simulation.**
- **For more information:**
  - **read the paper in the Proceedings**
  - **download our short course on Modeling and Simulation of Beam Control Systems, http://www.mza.com/doc/MZADEPSBCSMSC2004**
  - **or contact me, Steve Coy, coy@mza.com.**