Technique for Modeling Multi-Mode Stable-Resonator Laser using Multiple Iteration Averaging of the Intensity Profile

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Outline

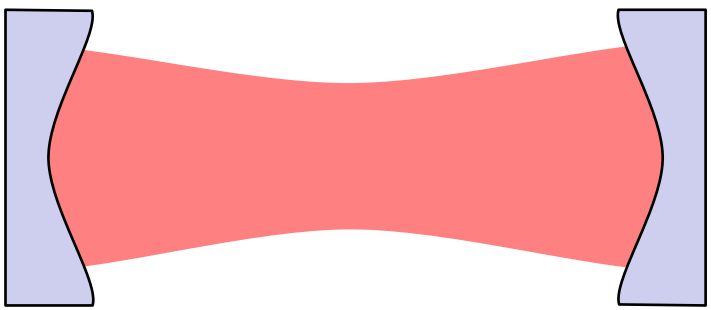
- Introduction & Motivation
- Choosing a Wave-Optics Mesh
  - Multi-Iteration Imaging
- Stable Laser Resonator Modeling Results
- Conclusions

Introduction and Motivation

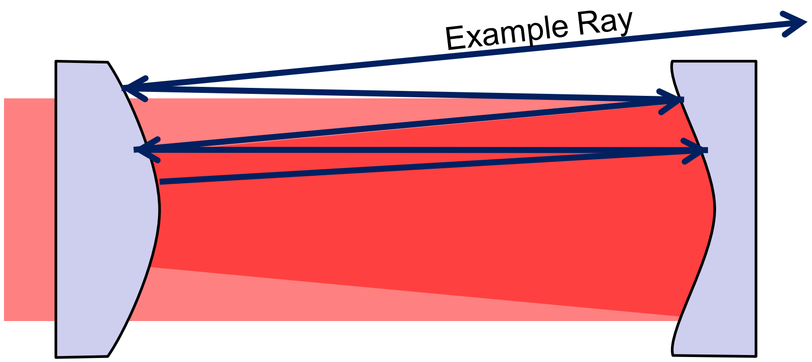
- When evaluating a new laser gain medium, it is common to build a multi-mode stable resonator around the gain medium to demonstrate maximum extraction.
- Modeling multi-mode lasers is difficult because
  - The modes tend to interact with the saturable gain medium and create modeling instabilities and
  - The mesh requirements for a multi-mode stable resonator are very resource intensive.

Laser Resonator Architectures: Stable vs. Unstable

Stable



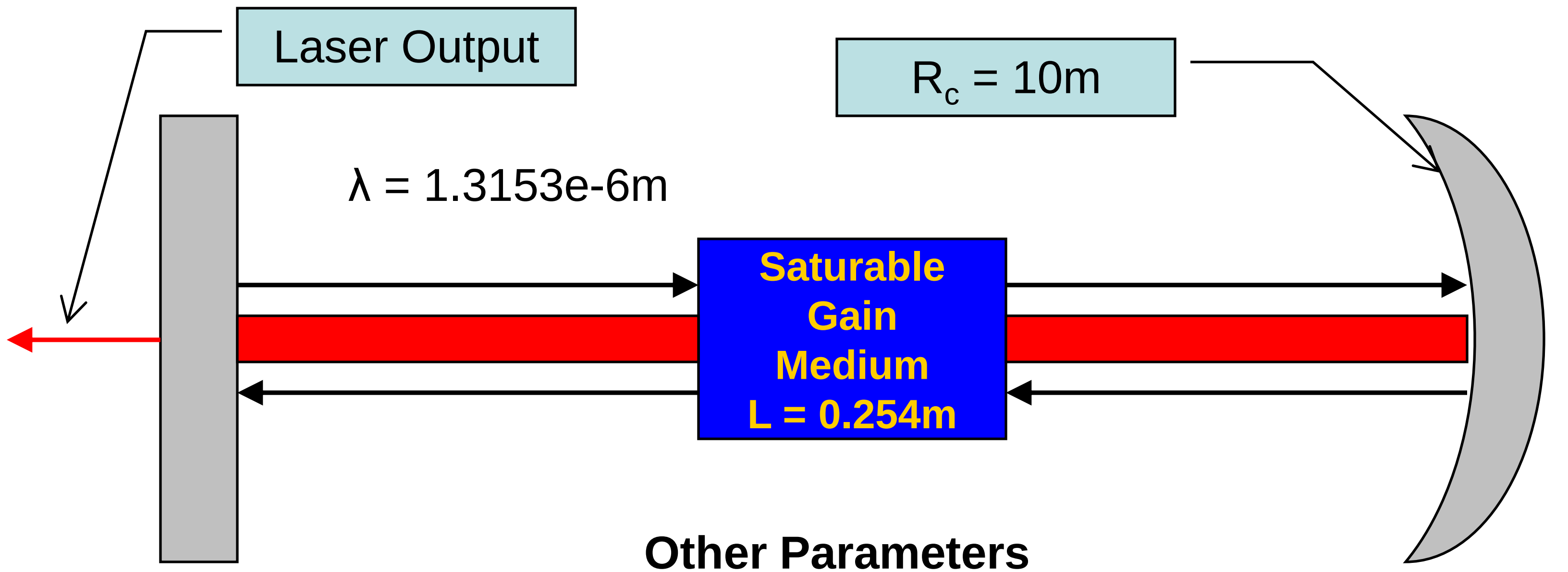
Unstable



Rays captured by a stable resonator will never escape geometrically.

All rays launched in an unstable resonator (except the on-axis ray) will eventually escape from the resonator.

RADICL Resonator



Flat Mirror

Curved Mirror

Other Parameters

$L_{resonator}$ = 0.8 m

D_{ap} = 0.06 m (mirrors and gain)

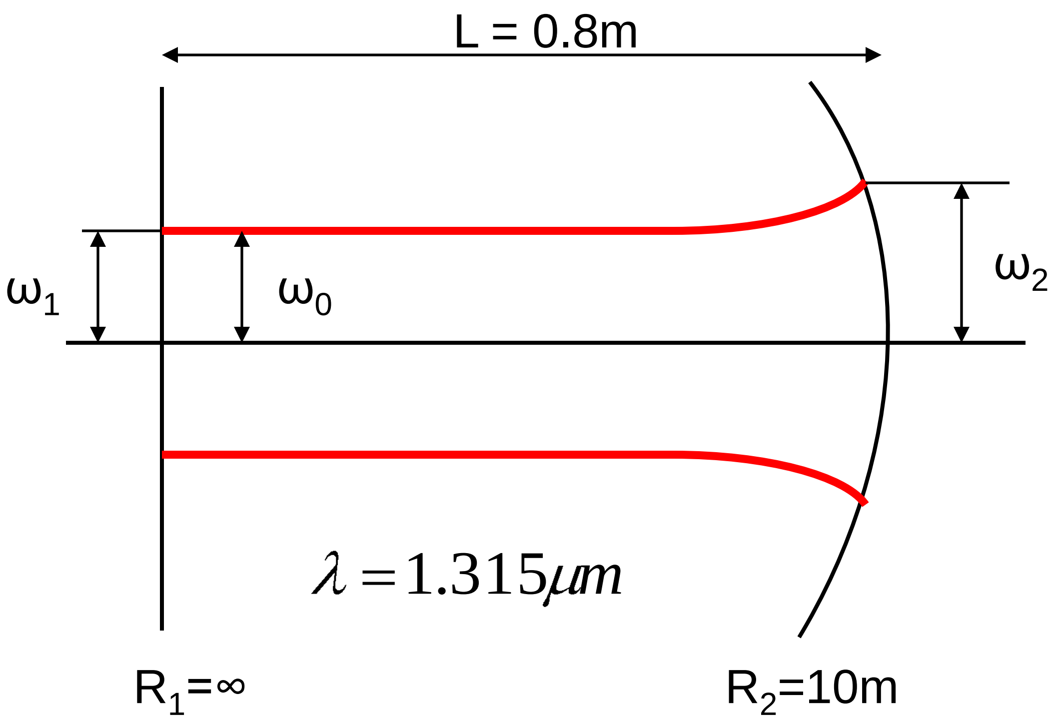
g_{ss} = 0.19435 %/cm

I_{sat} = 6.735 kW/cm^2

Data source:
Eppard, M., McGrory, W., and Applebaum, M. "The Effects of Water-Vapor Condensation and Surface Catalysis on COIL Performance", AIAA Paper No. 2002-2132, May 2002.

Predicted Gaussian Radius

TEM00 Mode Radius for Plano Concave Resonator



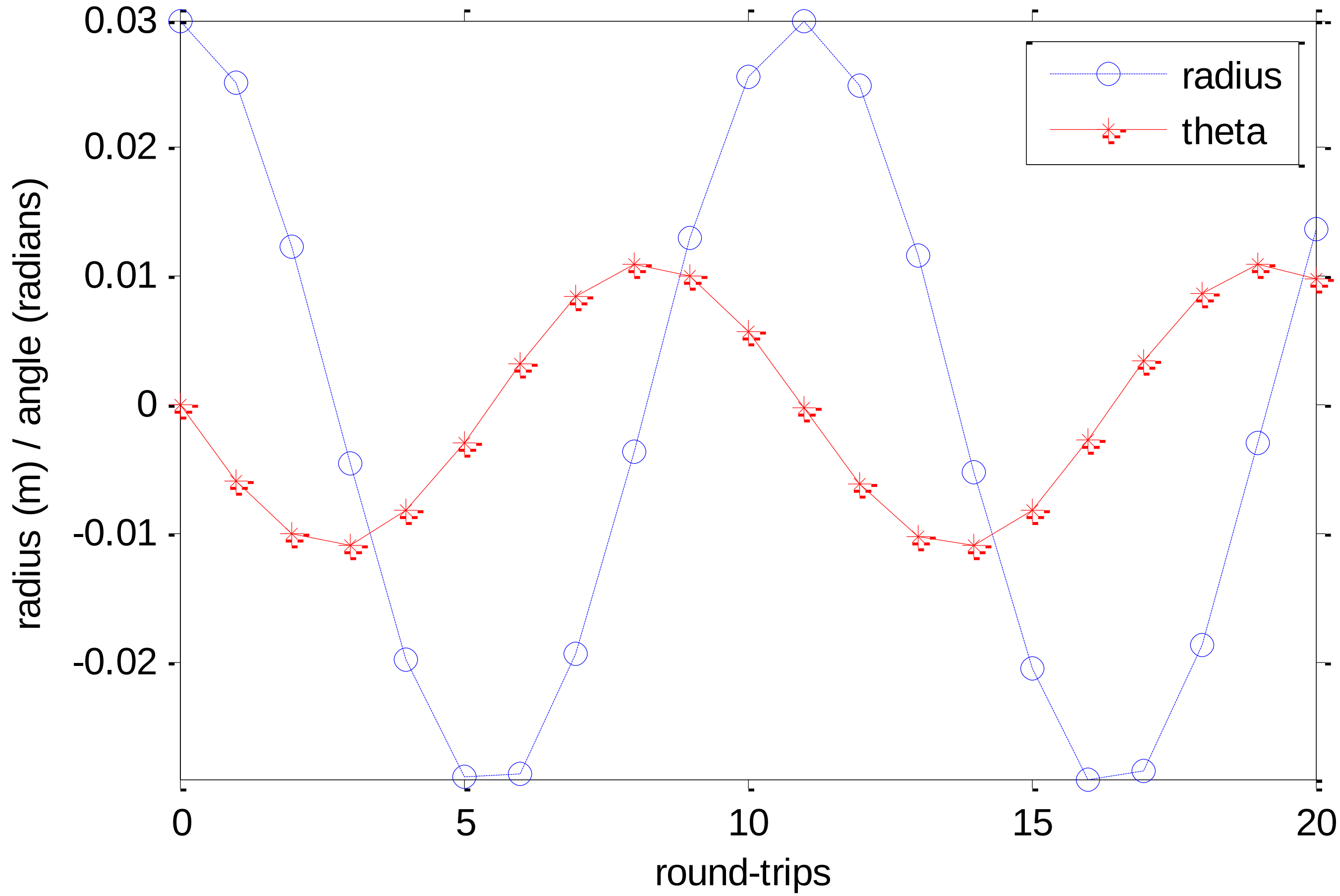
$$\omega_1^2 = \omega_0^2 = \frac{\lambda}{\pi}\sqrt{L(R_2 - L)}$$

$$\omega_2^2 = \frac{\lambda}{\pi} R_2 \sqrt{\frac{L}{R_2 - L}}$$

$$\omega_0 = \omega_1 = 1.0656mm$$

$$\omega_2 = 1.11mm$$

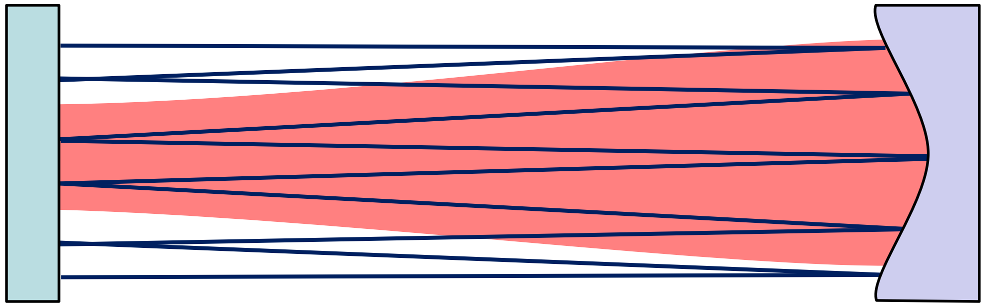
Rays and Angles Unwrapped



Graphical Method of Determining the Number of Round-Trips to Image

Number of Round-Trips to Image/Repeat

Plano-Concave Stable Resonator



Linearized Plano-Concave Stable Resonator

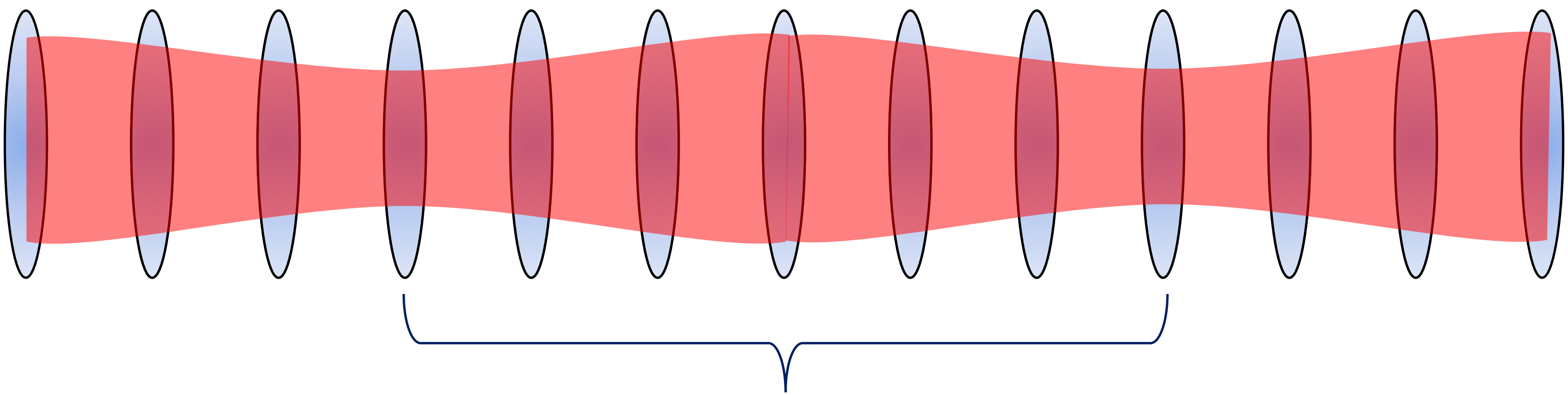


Image Planes

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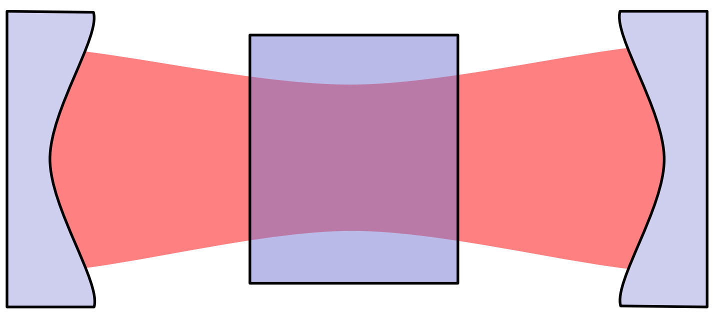
Stable Resonator With Gain



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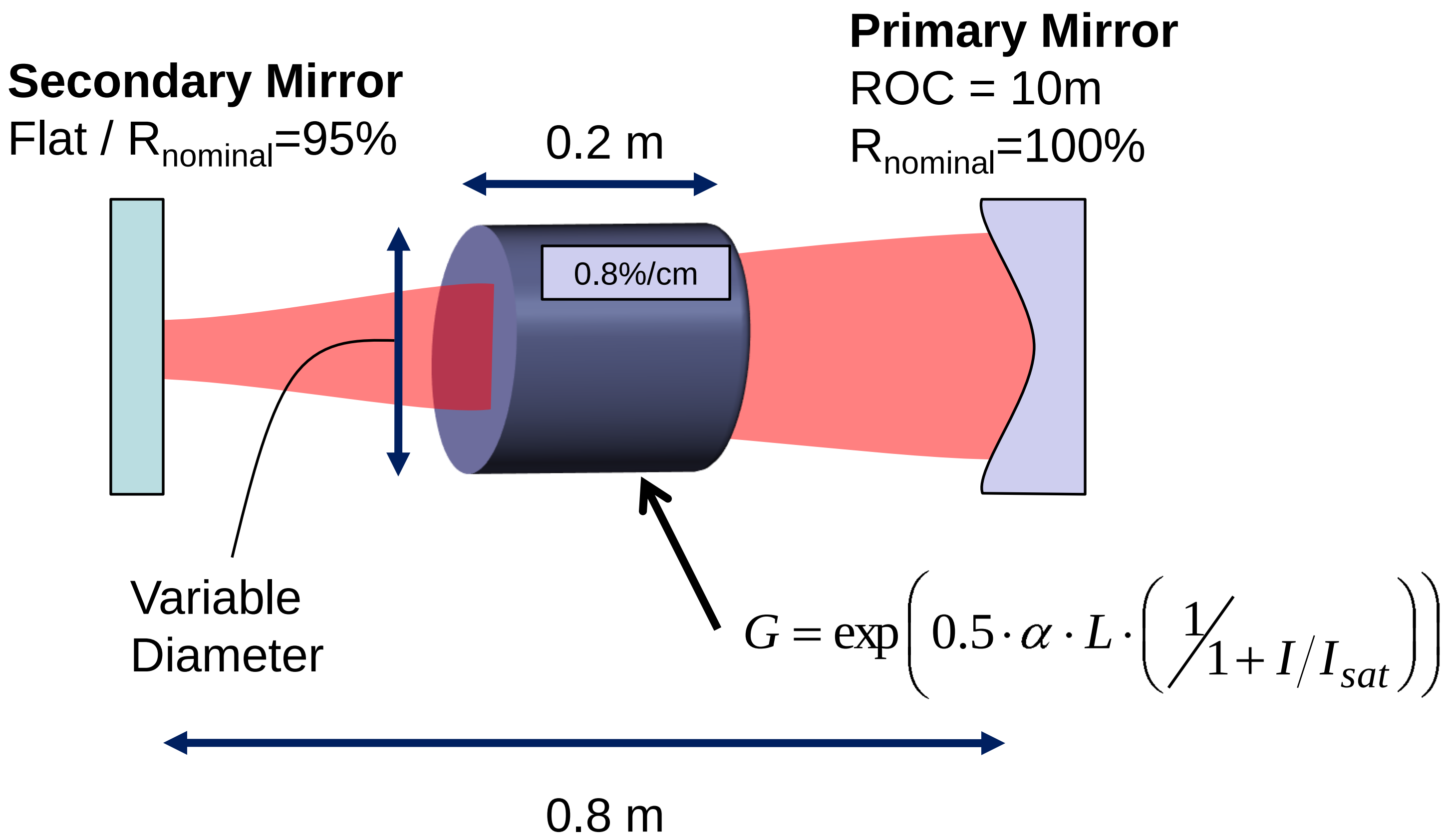
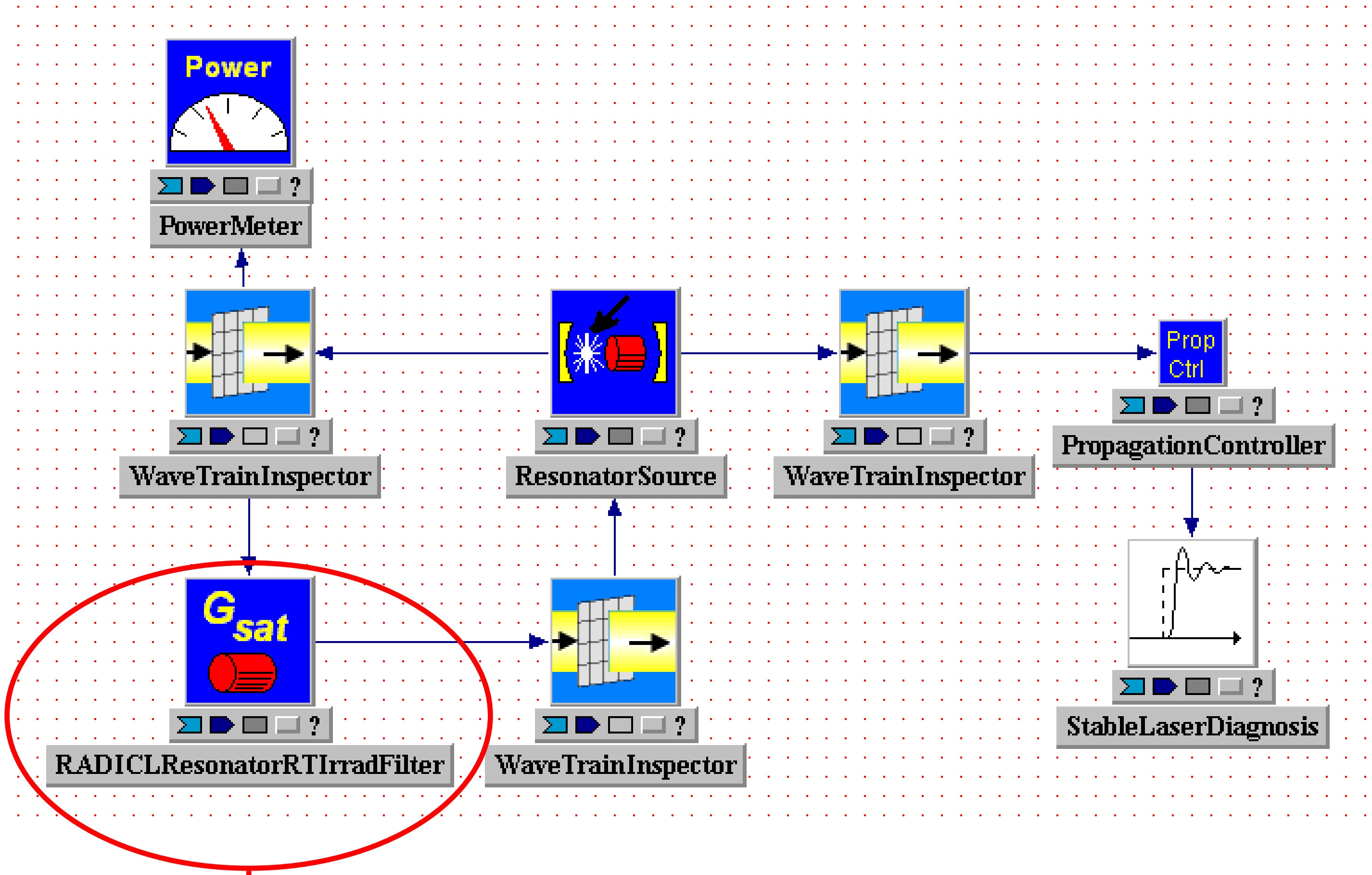
Example Laser Resonator Setup

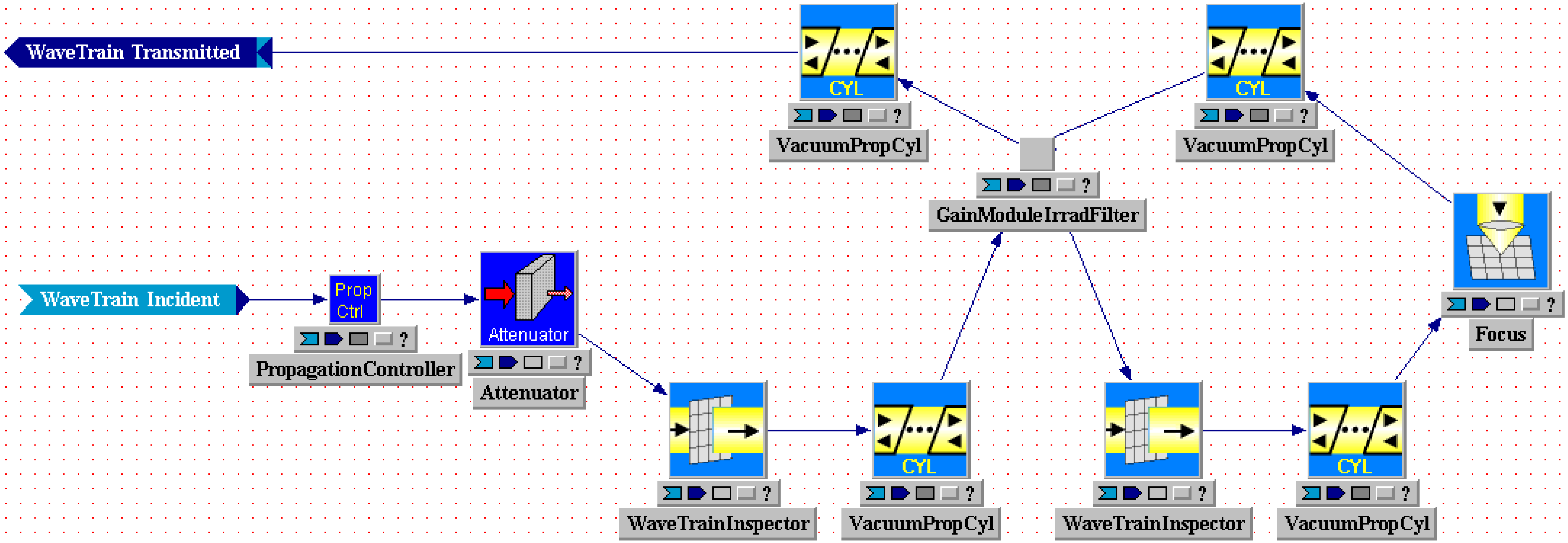


Wave Train Model



Next Slide…

Wave Train Model Of Laser Cavity



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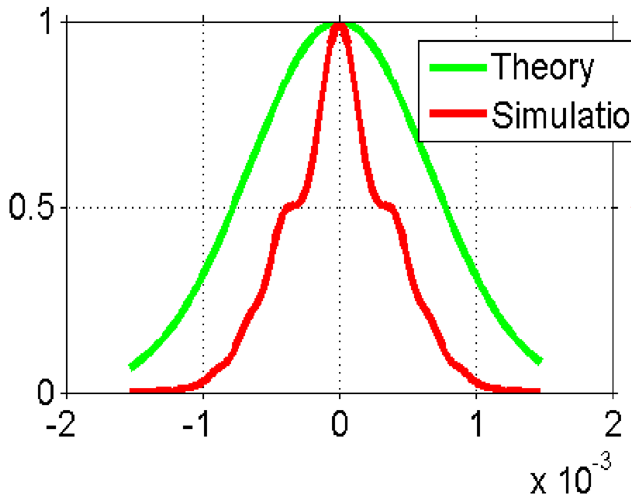
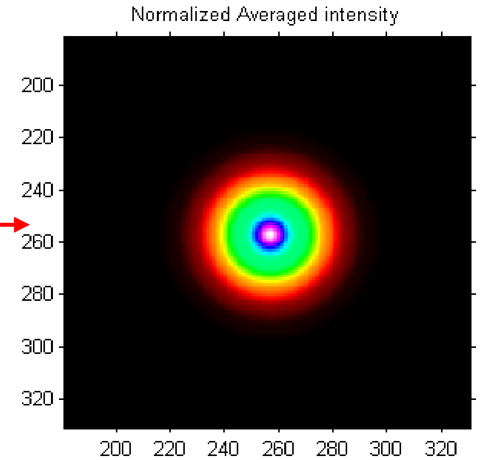
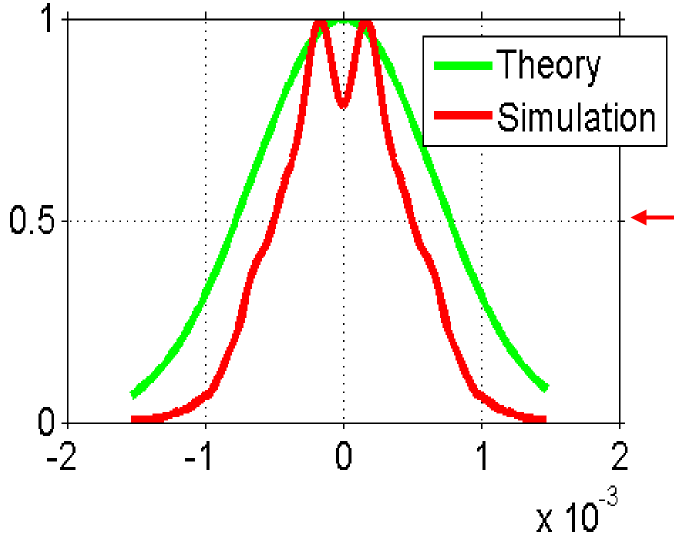
Initial Modeling Results
(No Intensity Averaging)

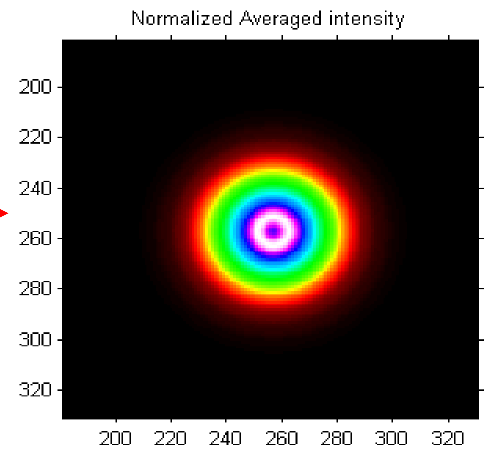
Intensity patterns for last frame and intensity plot along the central axis of Averaged intensity show the evolution of various modes with increase of aperture diameter





Aperture diameter = 2.0 mm

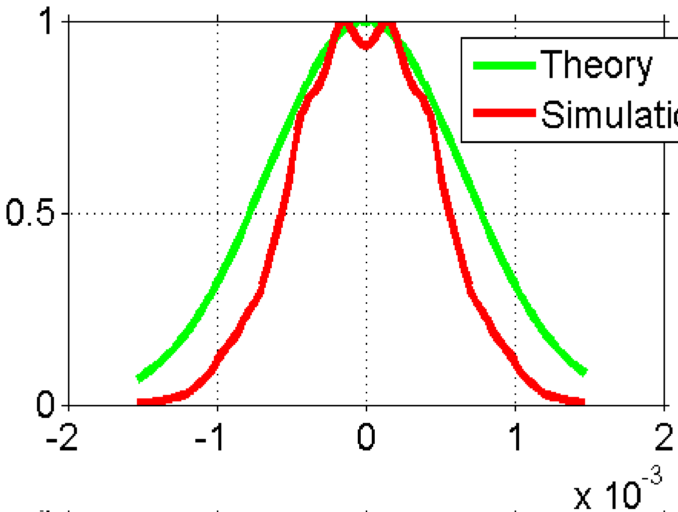


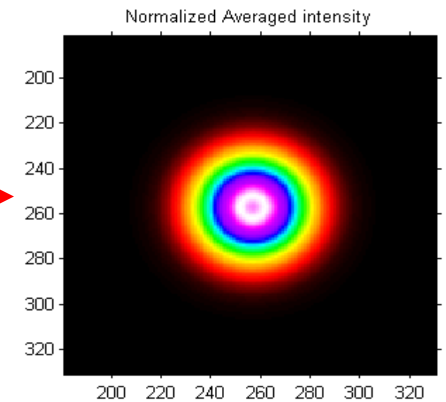


Aperture diameter = 2.5 mm

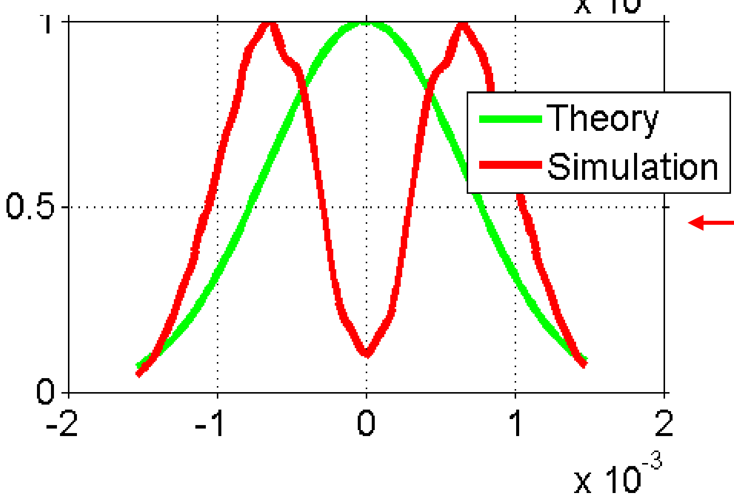
NOTE: Theory is TEM_{00} shape.

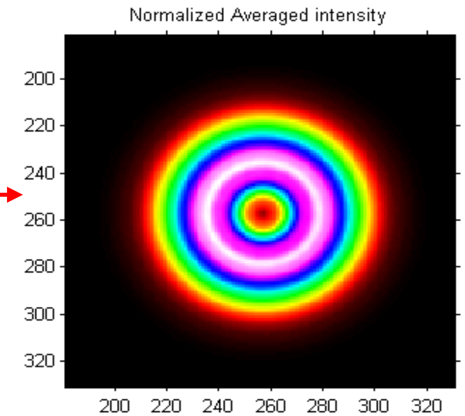
Intensity patterns for last frame and intensity plot along the central axis of Averaged intensity show the evolution of various modes with increase of aperture diameter





Aperture diameter = 3.0 mm

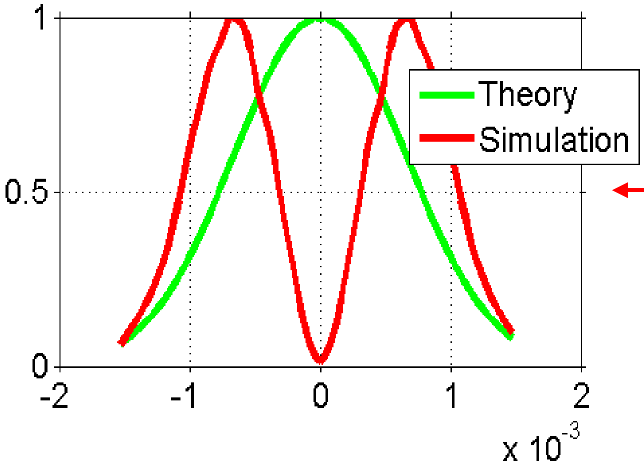


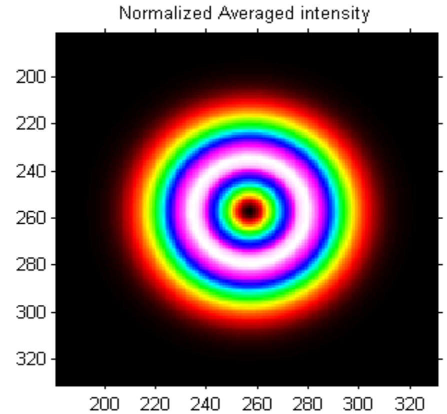


Aperture diameter = 3.5 mm

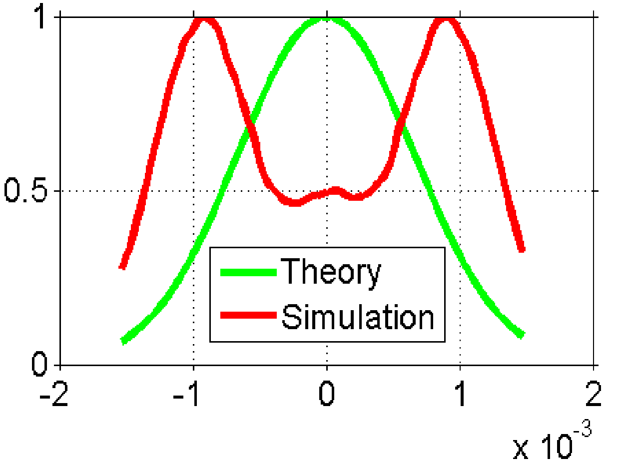
NOTE: Theory is TEM_{00} shape.

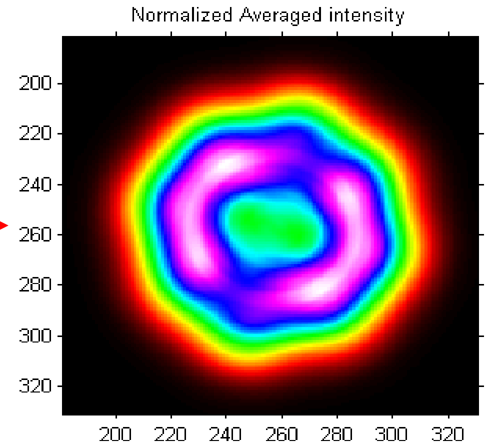
Intensity patterns for last frame and intensity plot along the central axis of Averaged intensity show the evolution of various modes with increase of aperture diameter





Aperture diameter = 4.0 mm

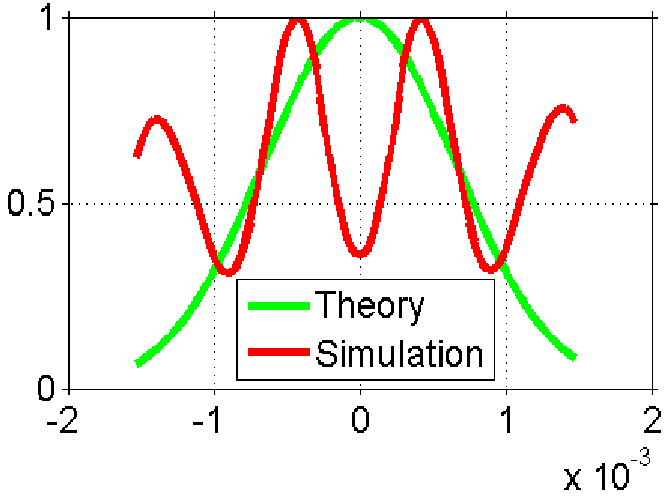


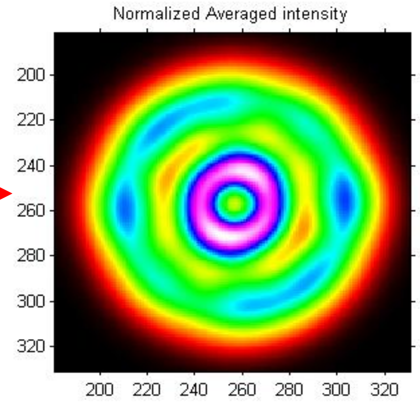


Aperture diameter = 4.5 mm

NOTE: Theory is TEM_{00} shape.

Intensity patterns for last frame and intensity plot along the central axis of Averaged intensity show the evolution of various modes with increase of aperture diameter

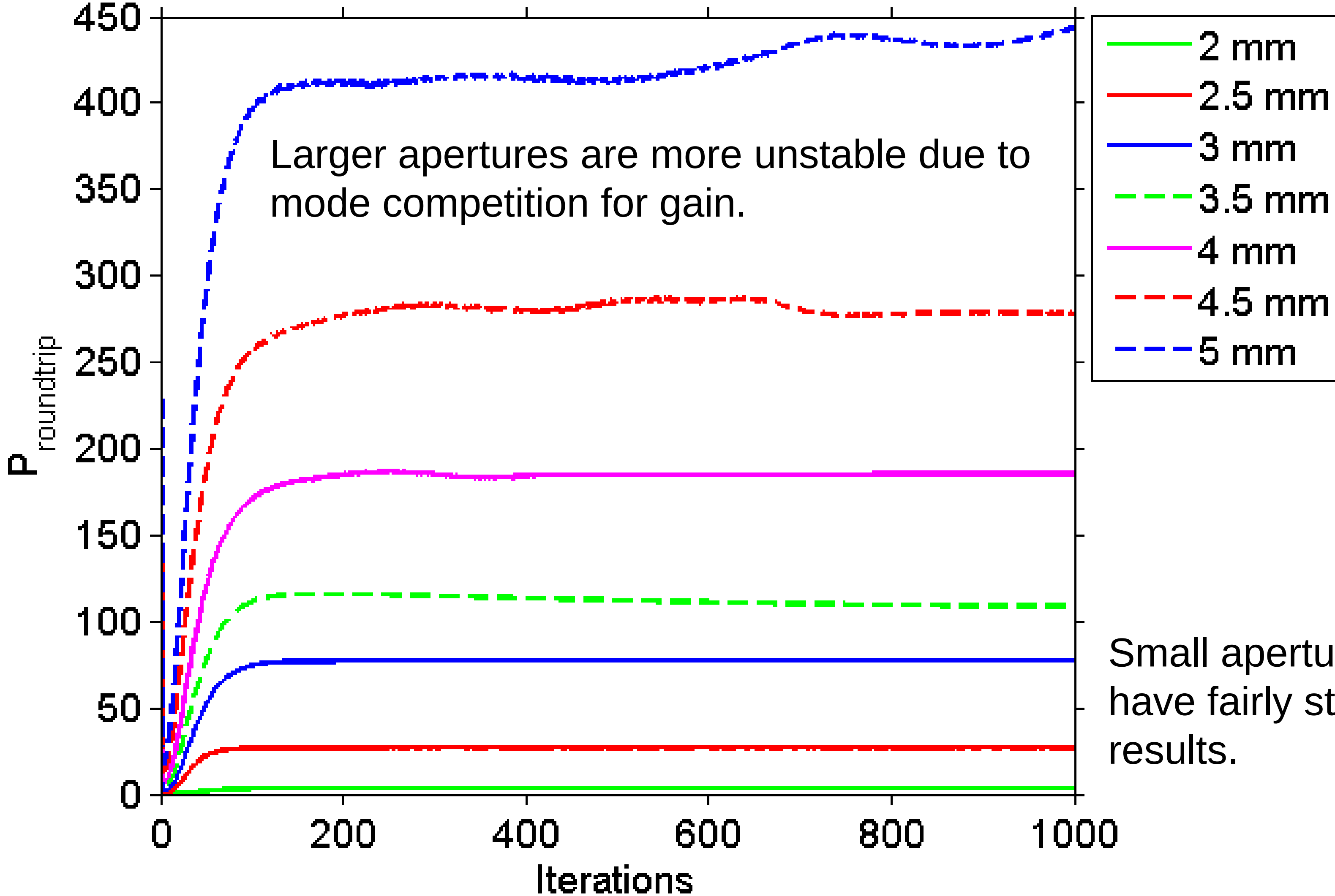




Aperture diameter = 5.0 mm

NOTE: Theory is TEM_{00} shape.

Output Power

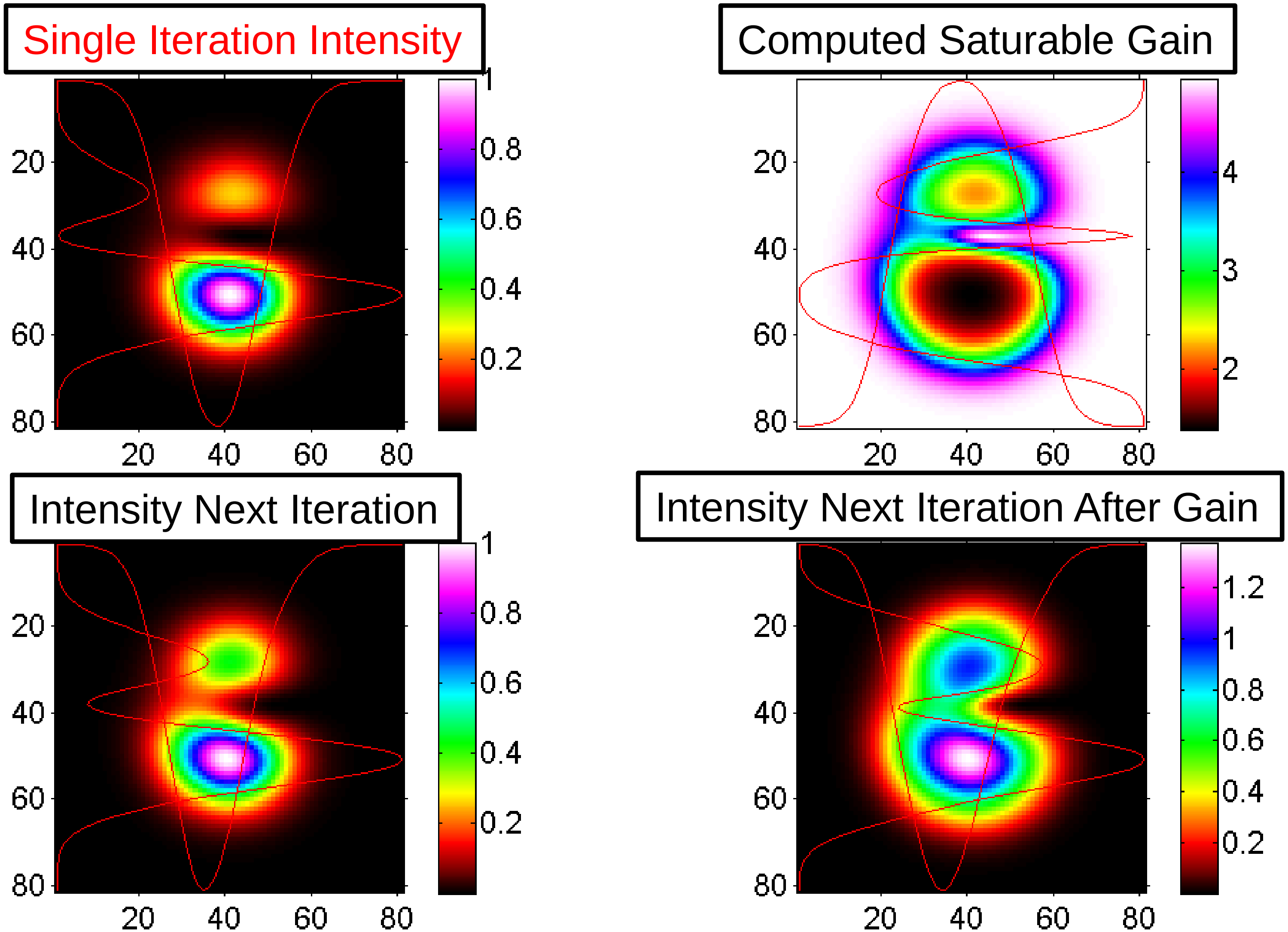


Small apertures have fairly stable results.

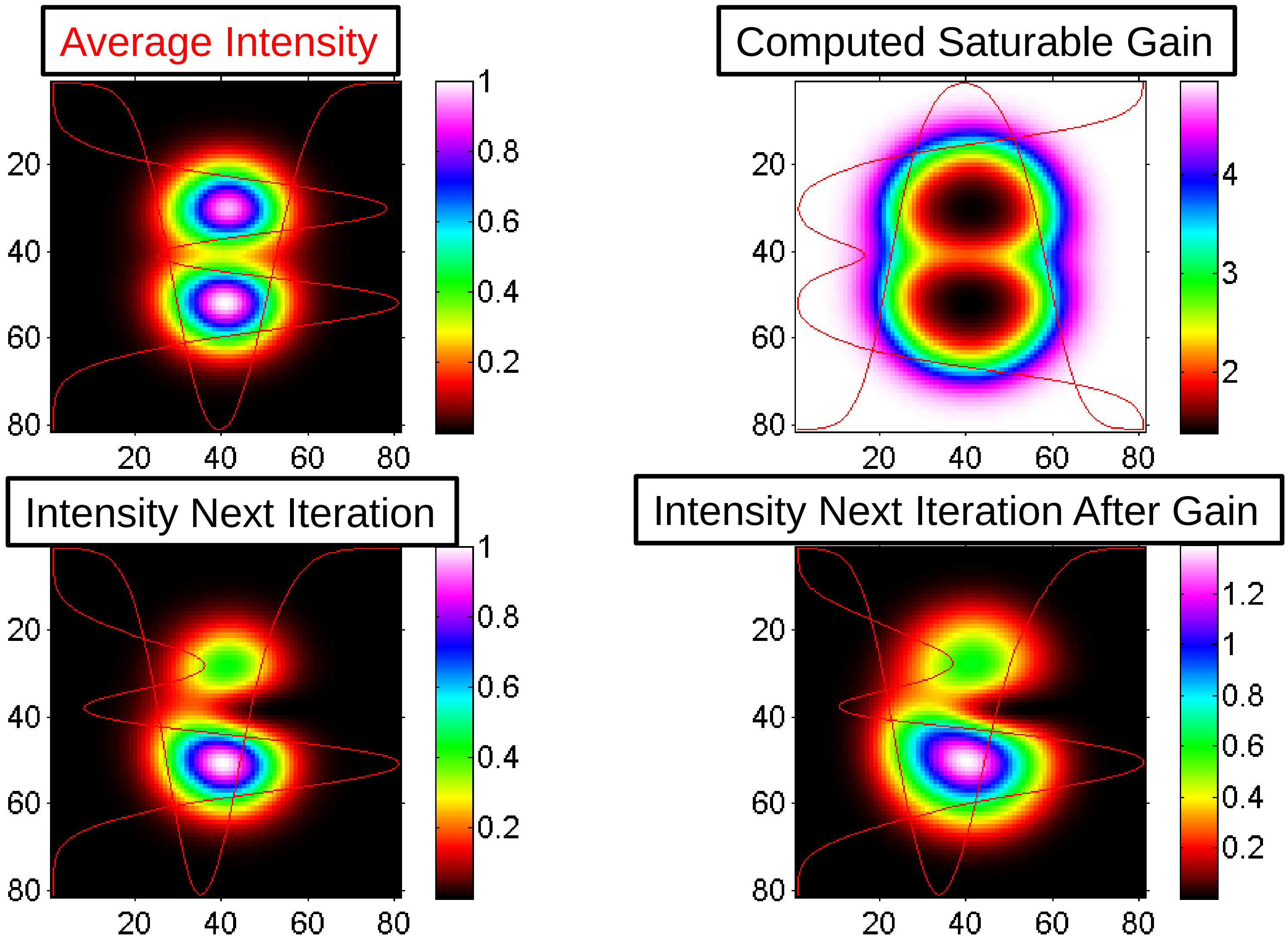
Results Analysis

- We found that by only using one iteration to calculate the intensity going into the saturable gain, that intensity profile was being “printed” onto the next iteration.
- We needed some way to stabilize the intensity being used for the saturable gain calculation, so we tried adding the ability to average the intensity over a user-specified number of previous iterations.
- Physical Justification: We are modeling the CW resonator with a single plane in the optical axis. Averaging allows us to include effects from other samples in time to add effective resolution along the optical axis.

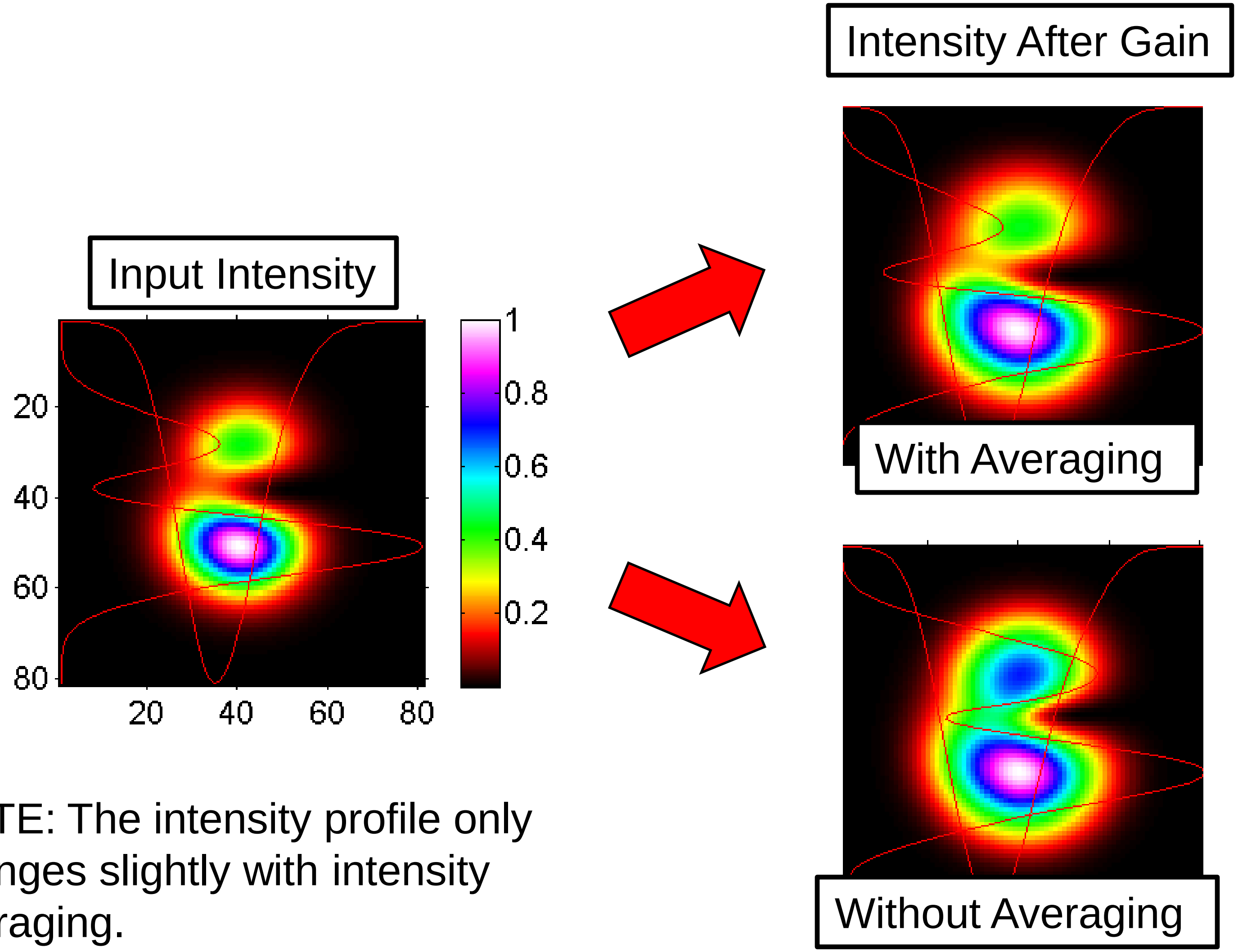
Example Iteration without Averaging



Example Iteration with Averaging



Comparison of Post-Gain Intensity with and without Averaging



NOTE: The intensity profile only changes slightly with intensity averaging.

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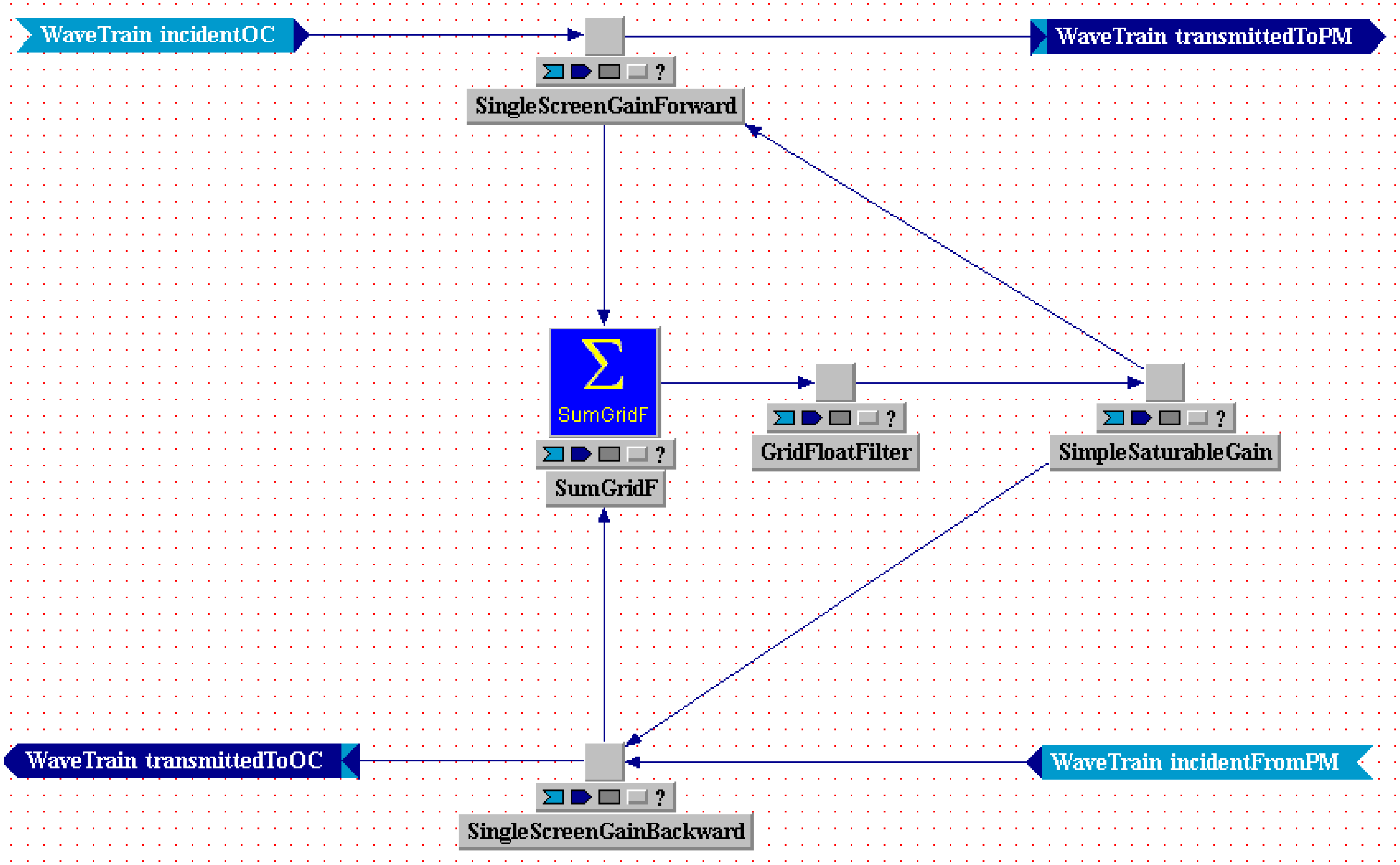
WT Model of Gain Medium



Description of the GridFloatFilter

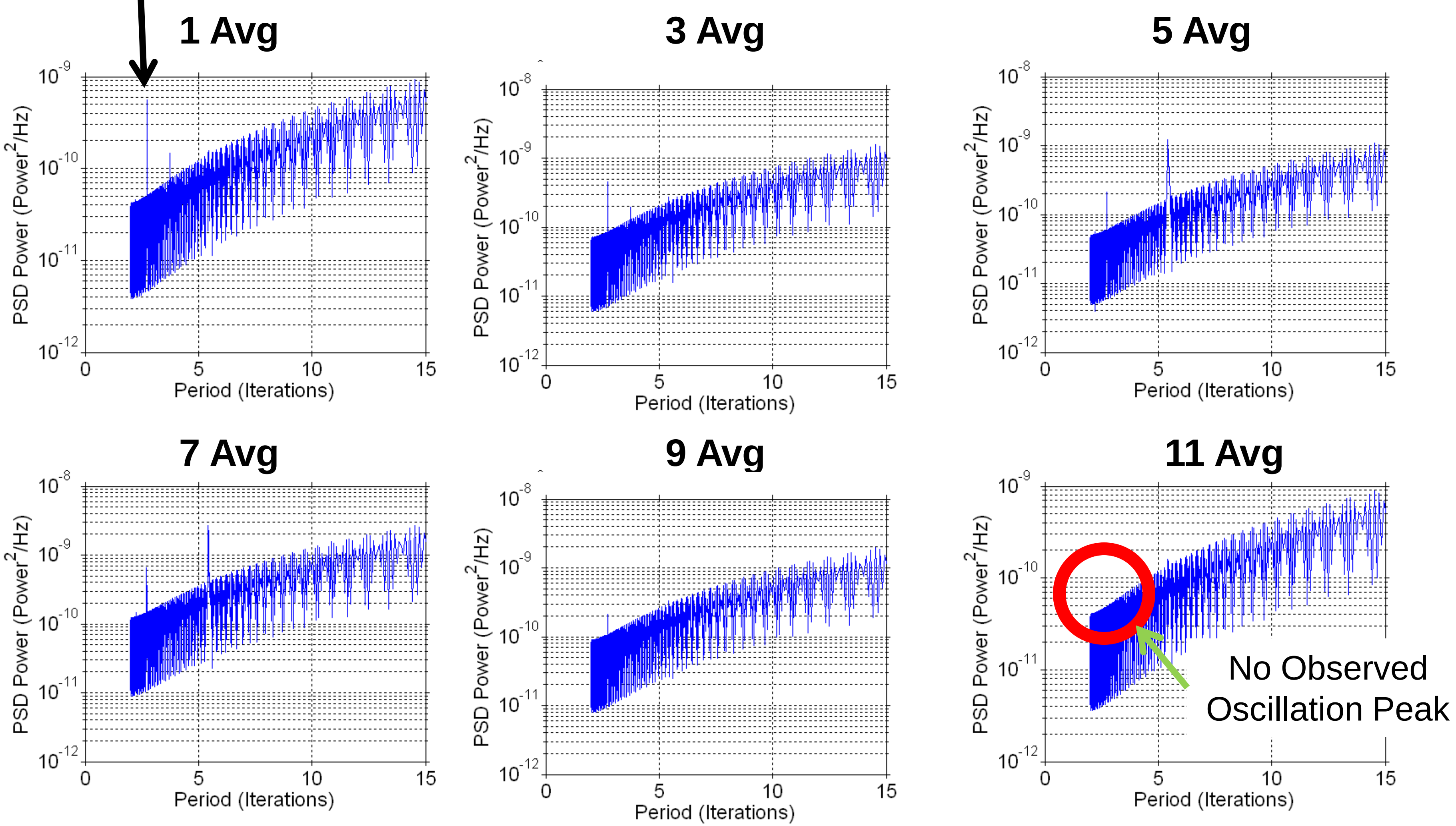
- We developed the GridFloatFilter to average a user-specified number of intensity frames together to get a more stable intensity profile.
- We found that the best results are achieved when the number of iterations over which to average is equal to the number of round-trips to image (see earlier derivation).
  - The original logic here was that the Fox & Li technique is modeling a single slice of a continuum of fields and by averaging we can take this into account.

Model Results for 11-Frame Intensity Averaging

1000 Iterations

Output Power PSD

2.73 iteration peak is exact ¼ harmonic of 10.955 oscillation



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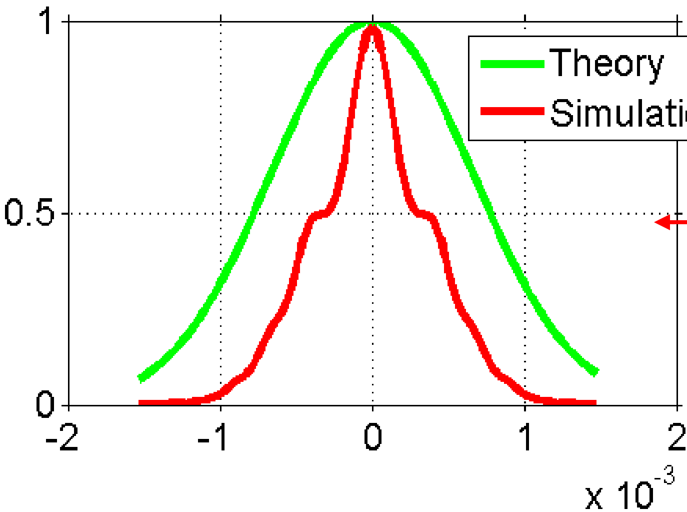
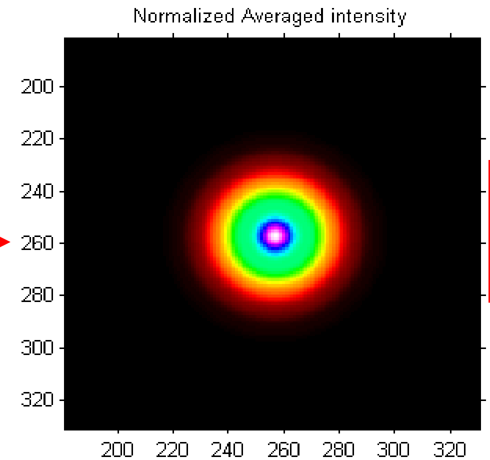
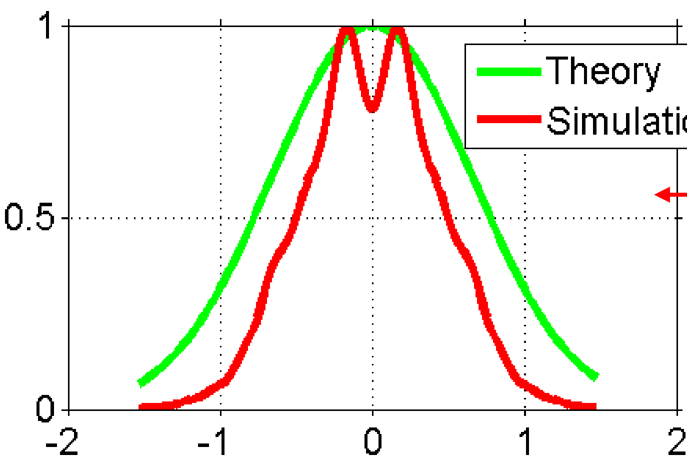
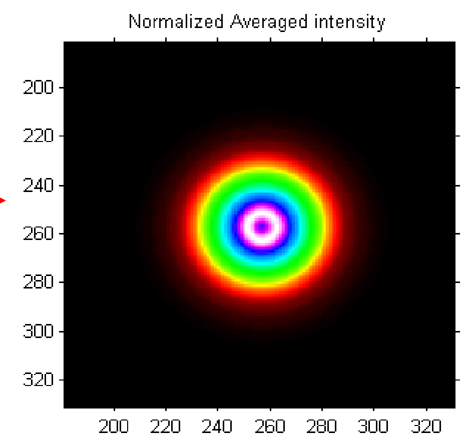
Small Aperture Results





Aperture diameter = 2.0 mm





Aperture diameter = 2.5 mm

NOTE: Theory is TEM_{00} shape

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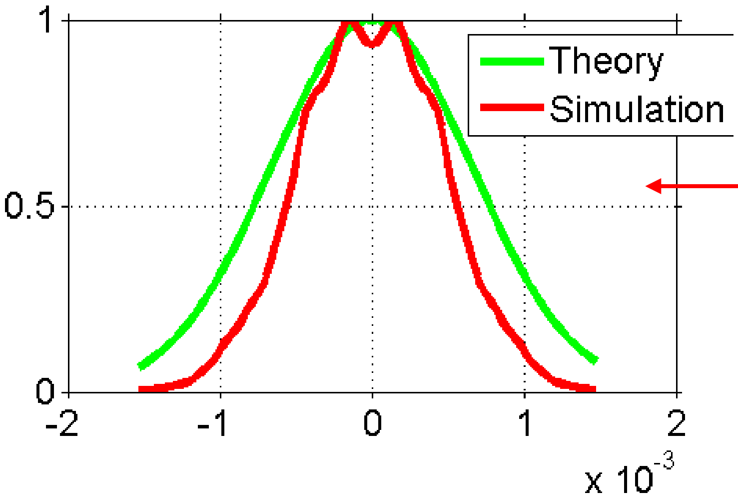
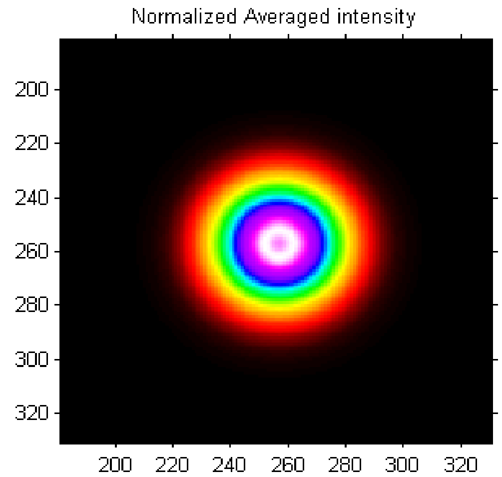
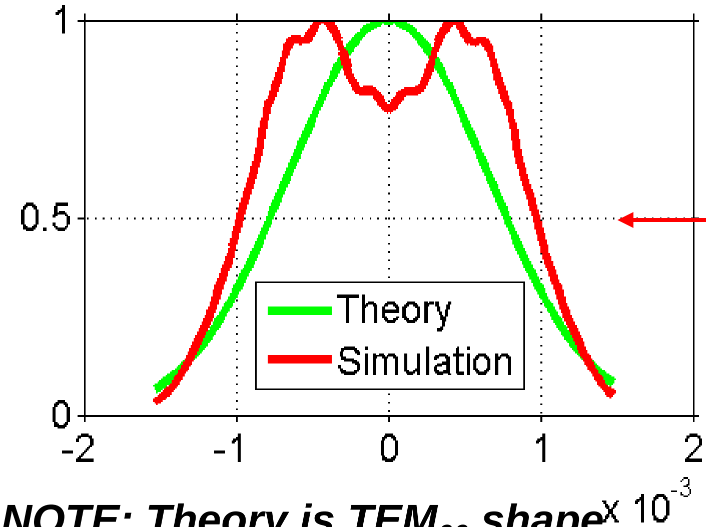
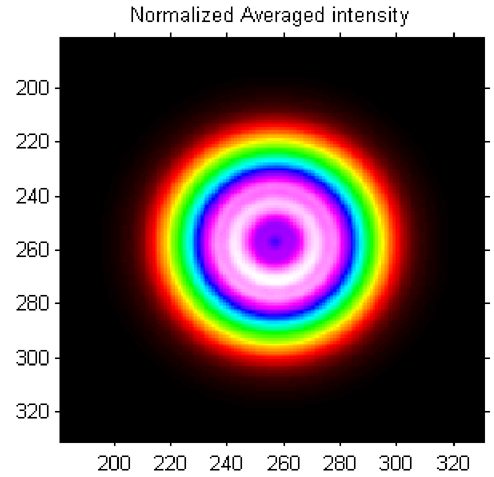
Medium Aperture Results





Aperture diameter = 3.0 mm





Aperture diameter = 3.5 mm

NOTE: Theory is TEM_{00} shape.

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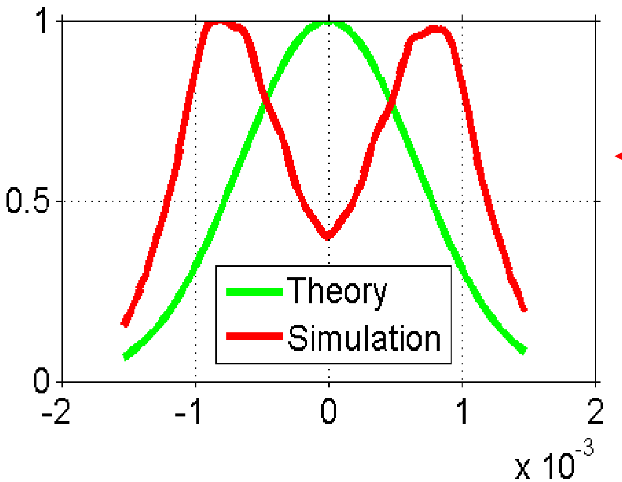
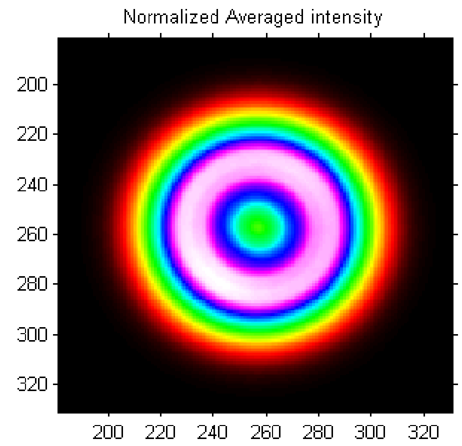
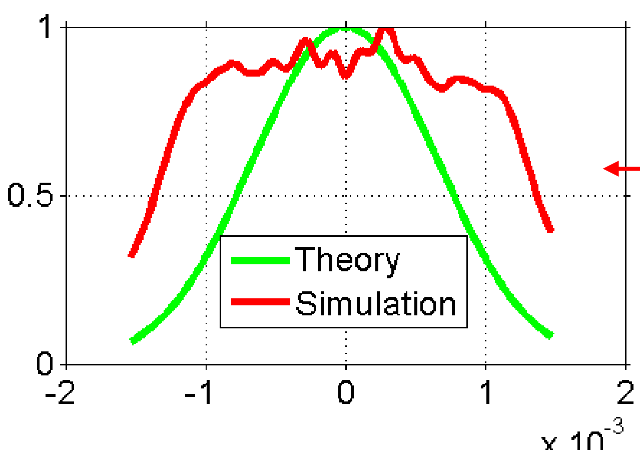
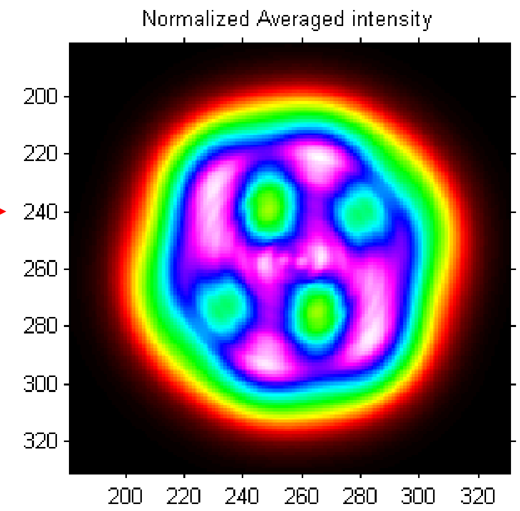
Larger Aperture Results





Aperture diameter = 4.0 mm





Aperture diameter = 4.5 mm

NOTE: Theory is TEM_{00} shape.

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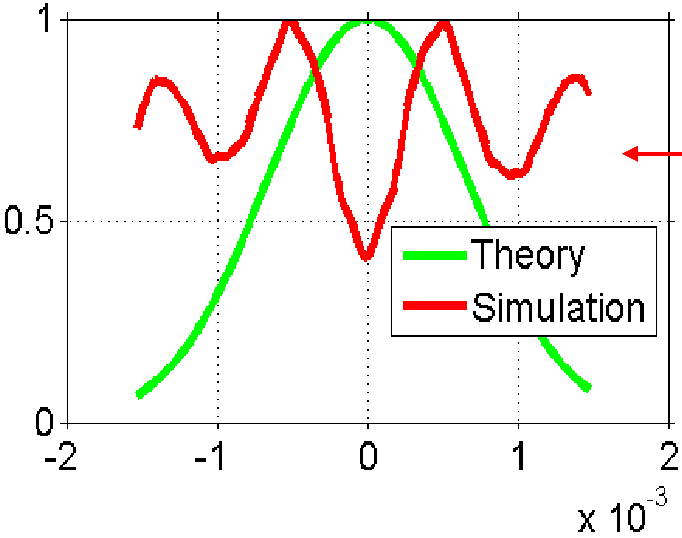
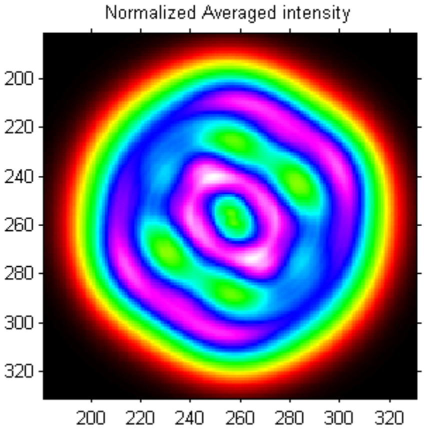
Largest Aperture Results





Aperture diameter = 5.0 mm

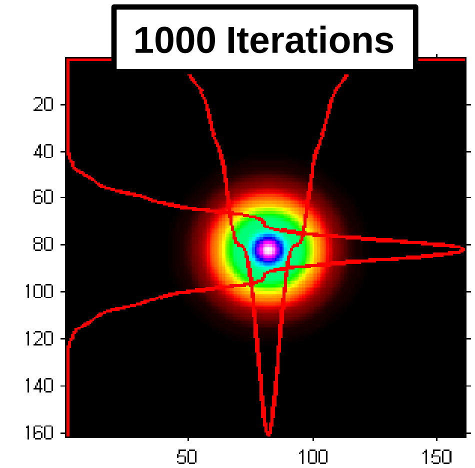
NOTE: Theory is TEM_{00} shape.

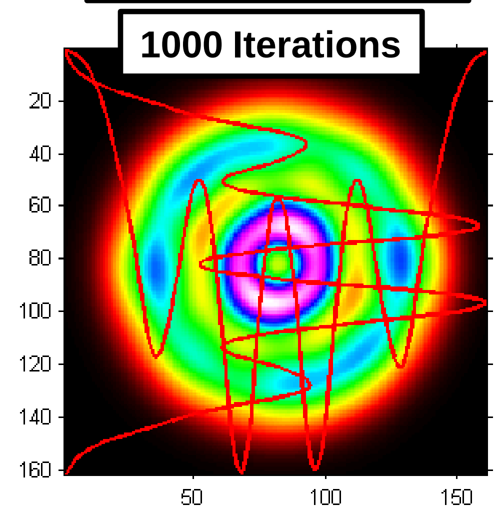
Comparison of Intensity Profiles

2-mm Aperture

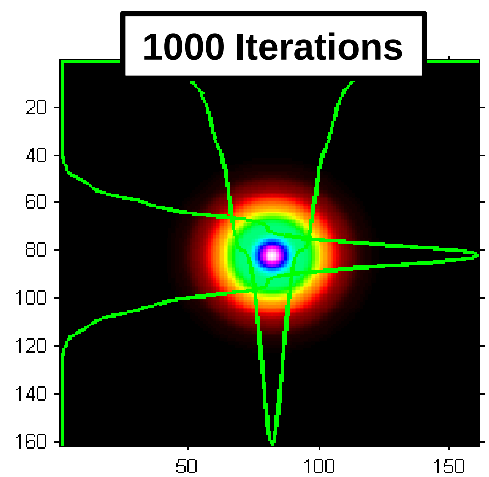
5-mm Aperture

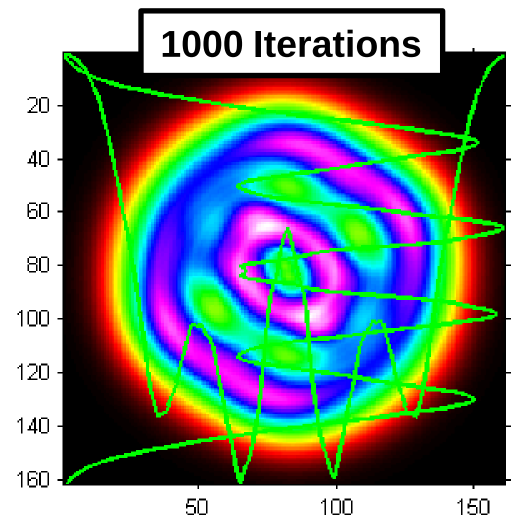
No Averaging

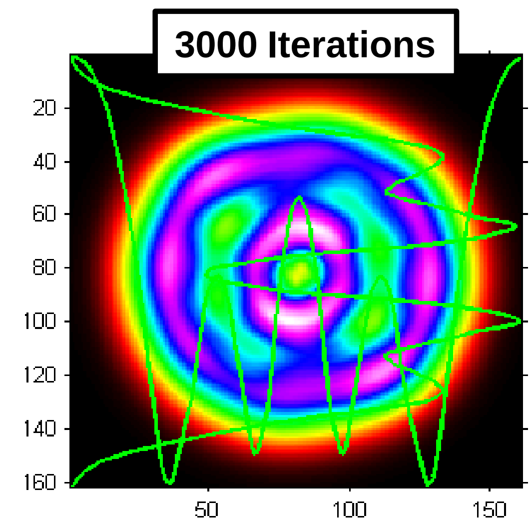




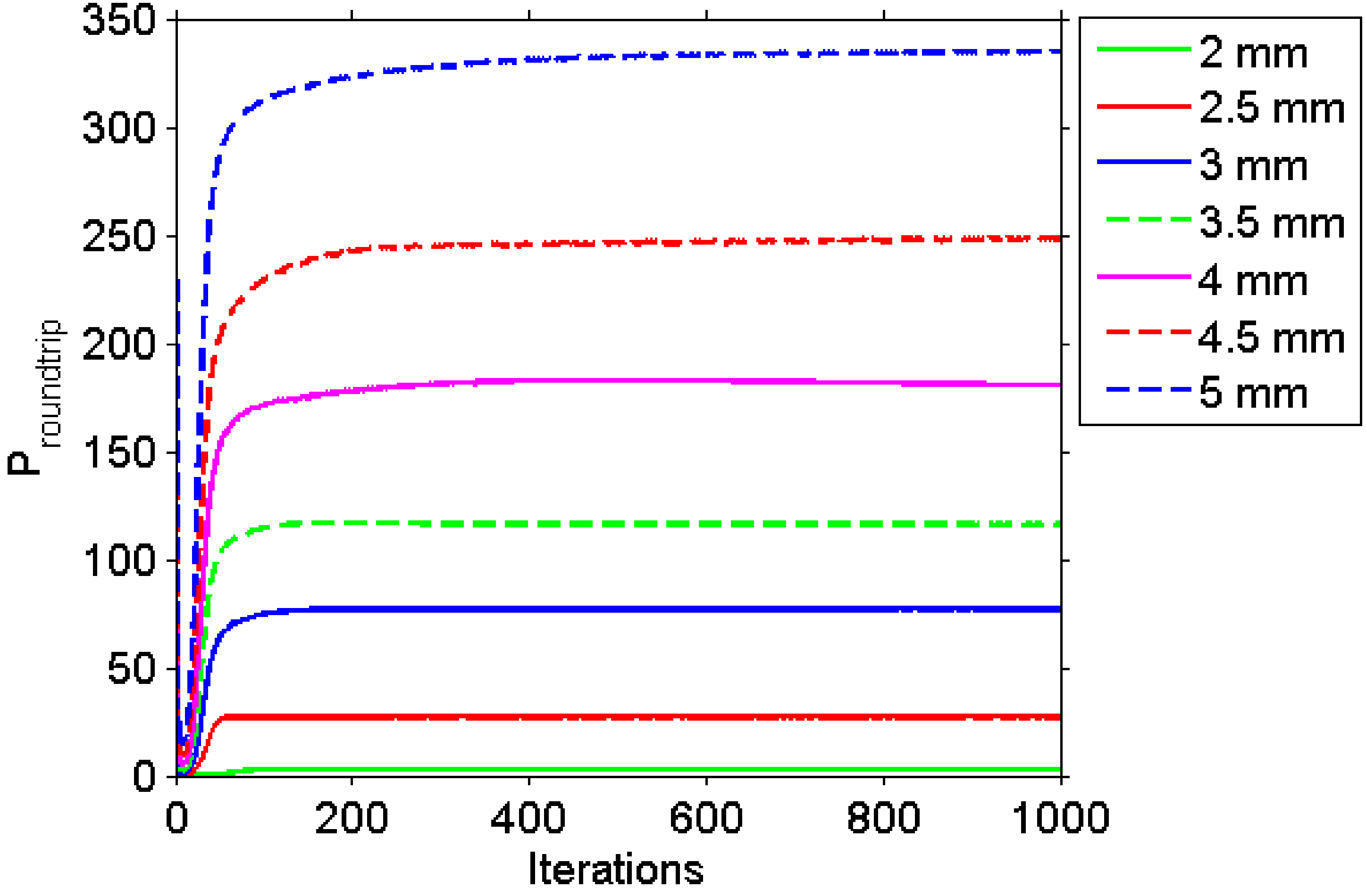
11-Frame Averaging







Output Power Analysis vs. Iteration



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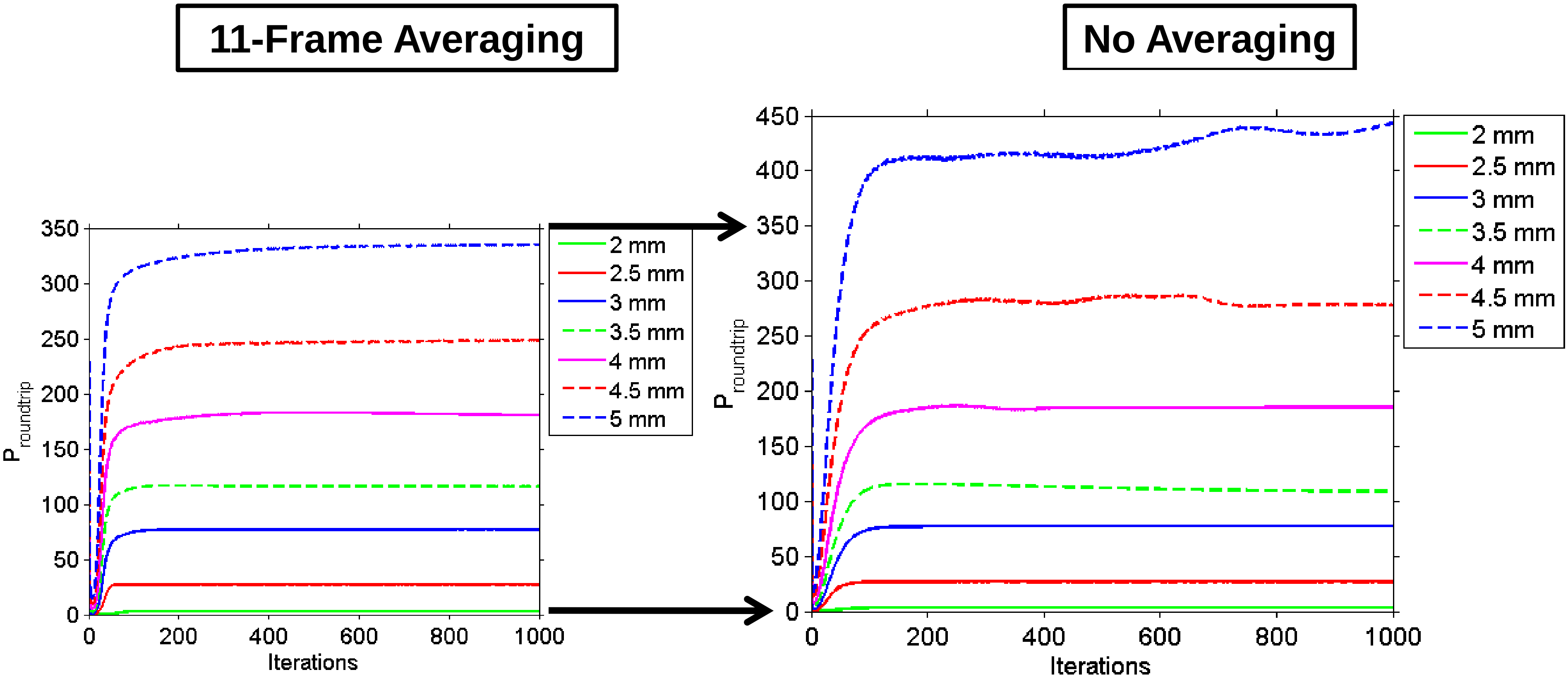
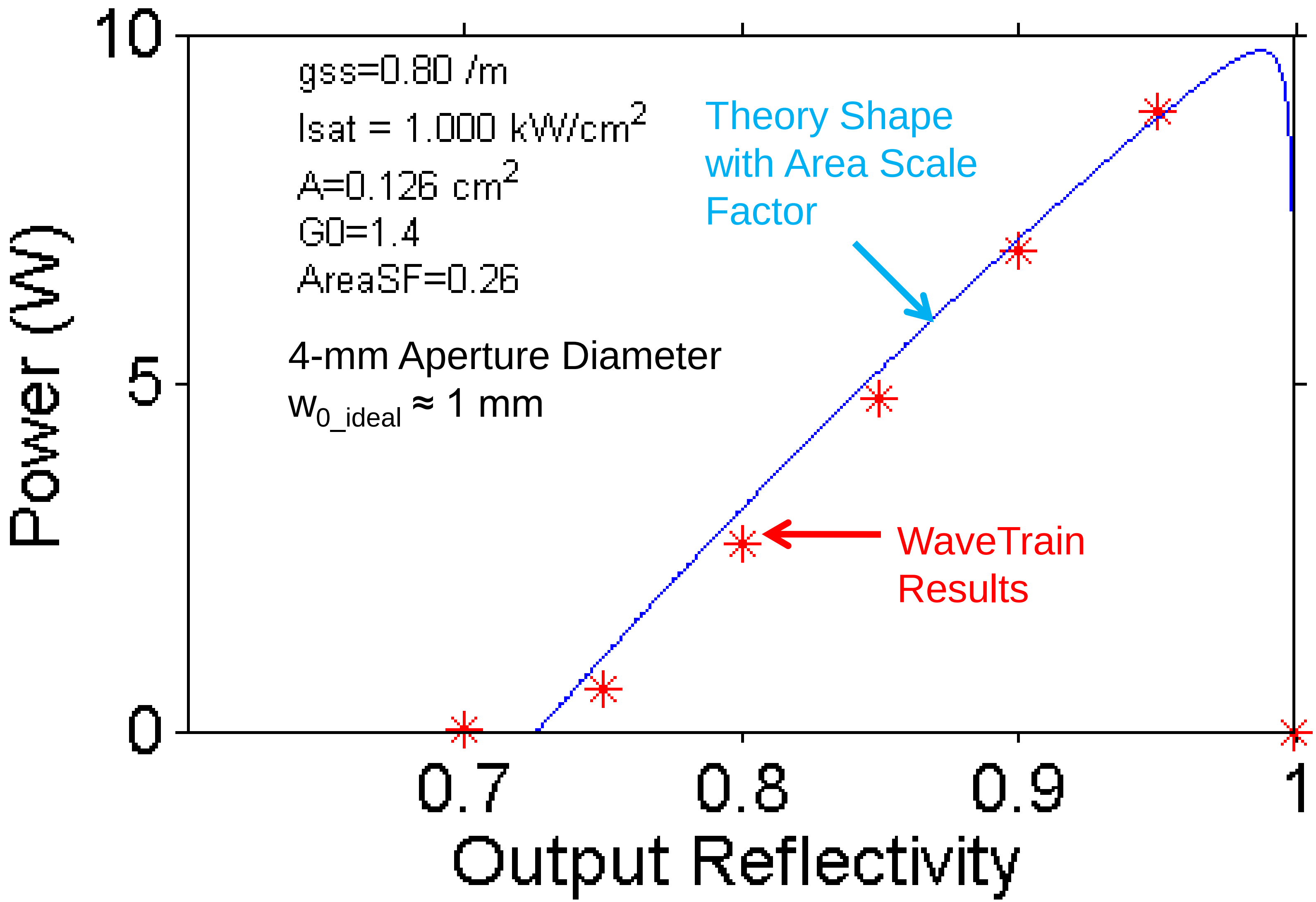
The averaging case converges more rapidly and to a steadier state in the presence of multiple transverse modes running.



Shape and power Level is comparable for small apertures, but significantly different for larger apertures. We are now trying to anchor to Rigrod power predictions.

Preliminary Rigrod Comparison



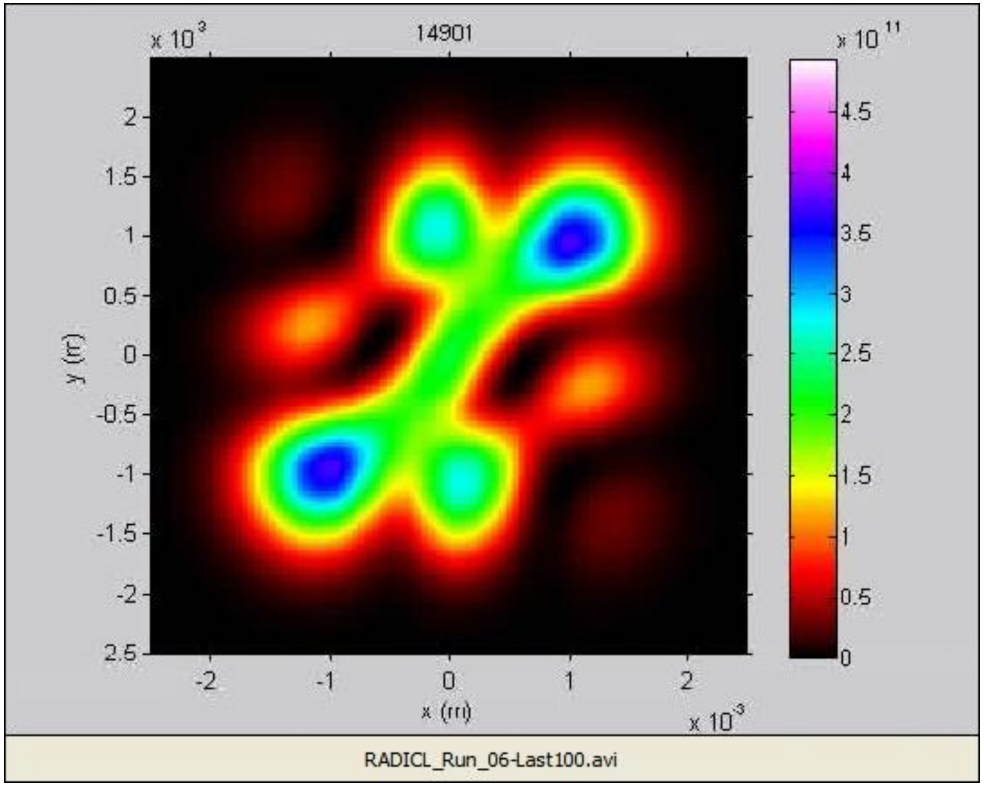
Comparison with and without Intensity Averaging

- Intensity averaging before the saturable gain causes a more stable output power vs. iteration and faster convergence
- Longer runs with averaging showed that the intensity profile is stable, but rotating.

Intensity Frame Averaging on the Small (5-mm) Square Aperture Laser Model

Reduced Aperture Beam Profile

- Movie shows last 100 irradiance frames
- Change in pattern over time is due to time-dependent phase differences between dominant HG modes of the cavity
- This time dependence is easily verified through a simple geometric ray analysis



Aperture Diameter = 5mm

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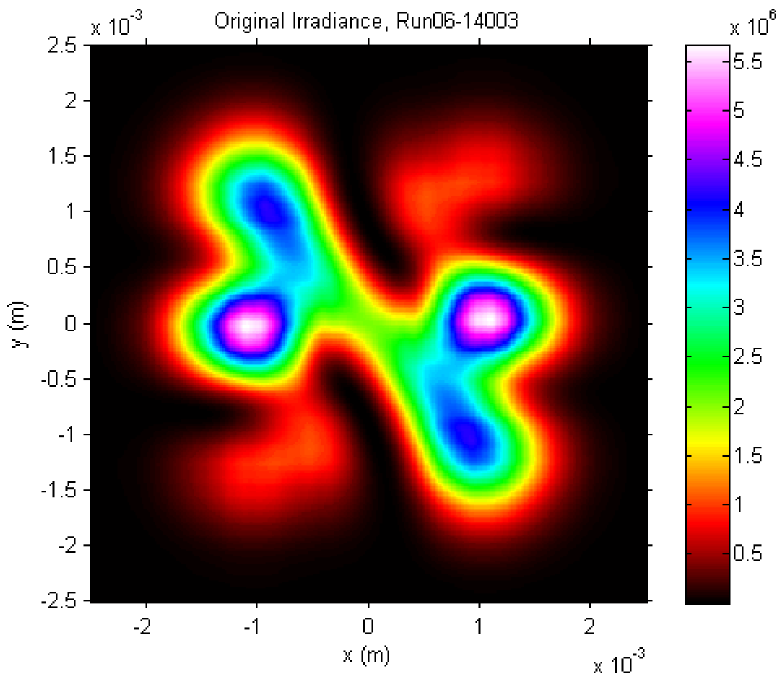
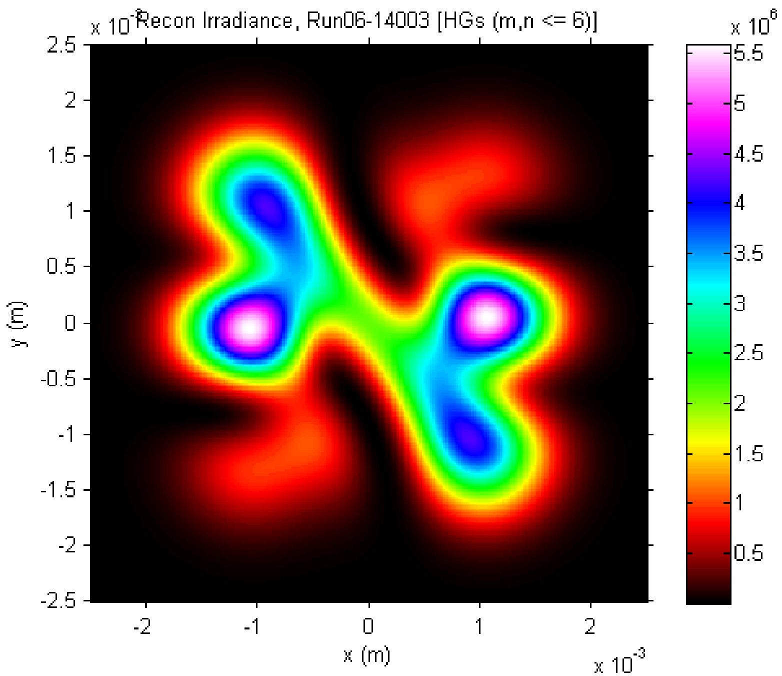
HG Decomposition – Accuracy

- Using theoretical waist, decompositions of the fields were performed, and show very good accuracy

Original Irradiance



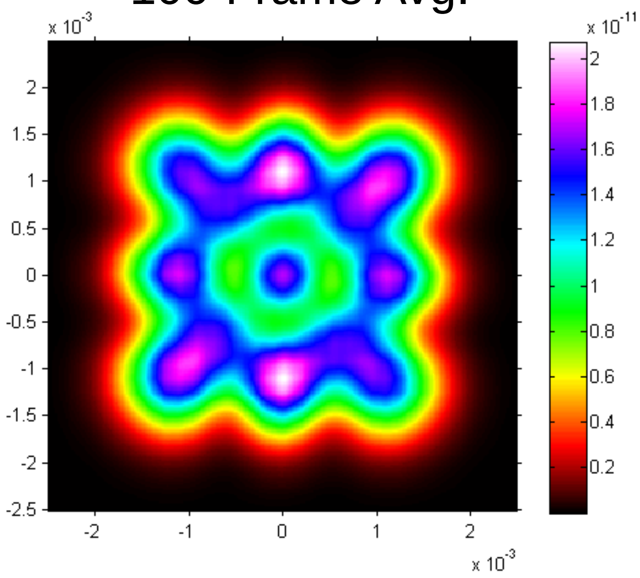
Reconstructed Irradiance



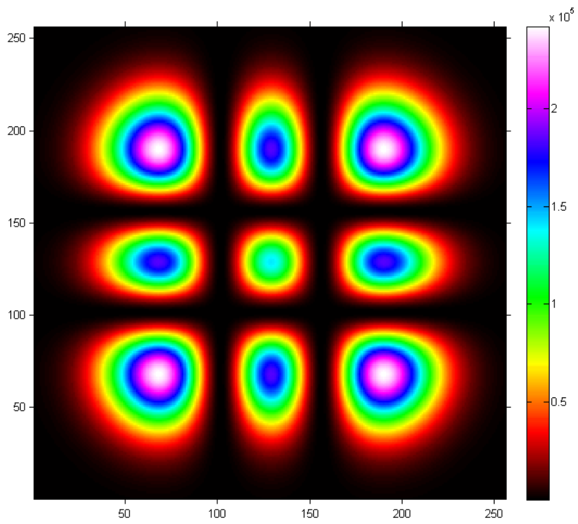
Beam Profile Decomposition

- In fact, the time-average of the last 100 irradiance frames has some characteristics of a HG22 mode:

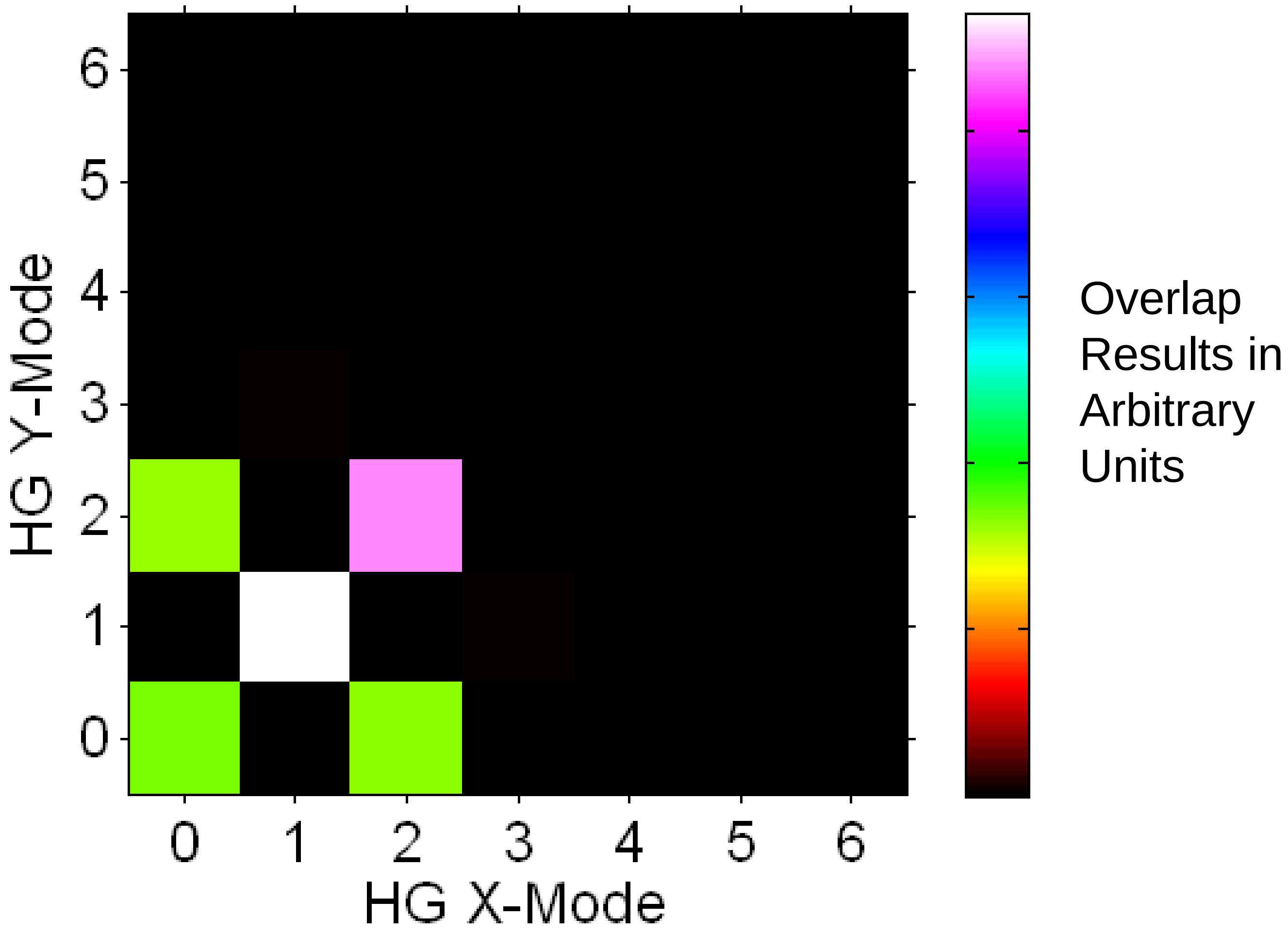
100-Frame Avg.



HG22



HG Decomposition



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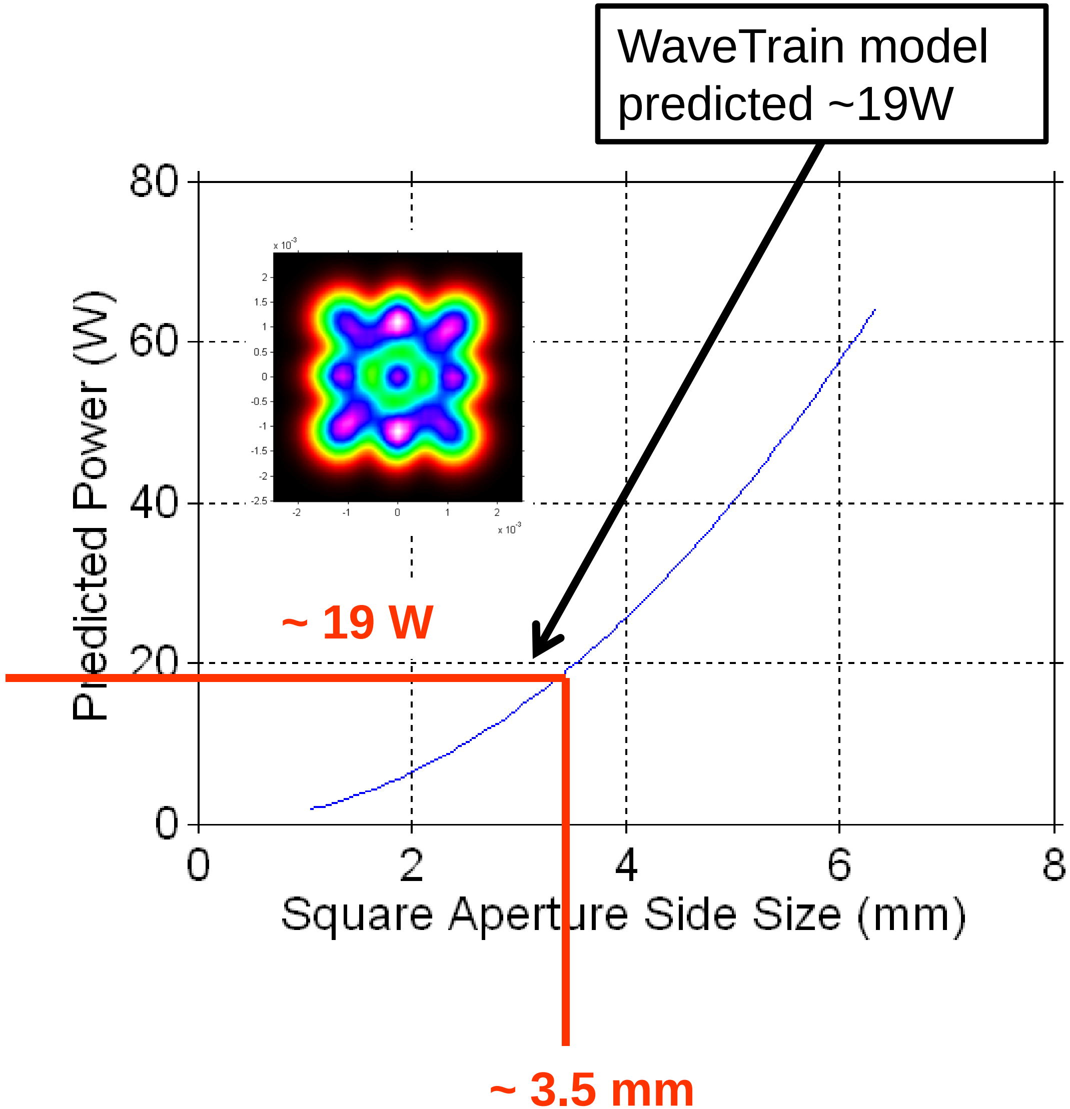
Rigrod Analysis of Power Level



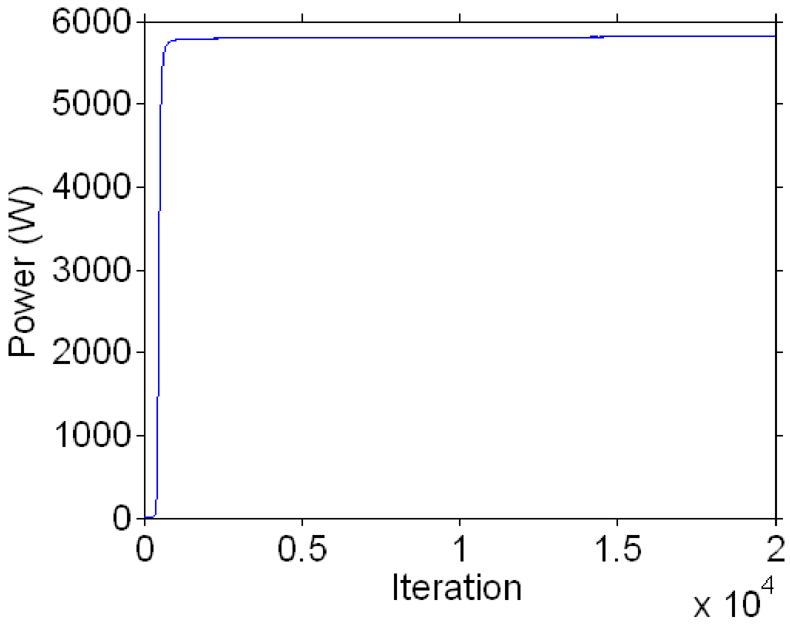
- Rigrod predicts power for "large output coupling" and for uniform field
- Thus we would expect a non-uniform field to have lower power (or equivalently a smaller "effective" aperture)
- I_Rigrod = 1.5974e+006 W/m^2
- Flattening simulated fields to I_Rigrod matches closely to 3.5 mm aperture

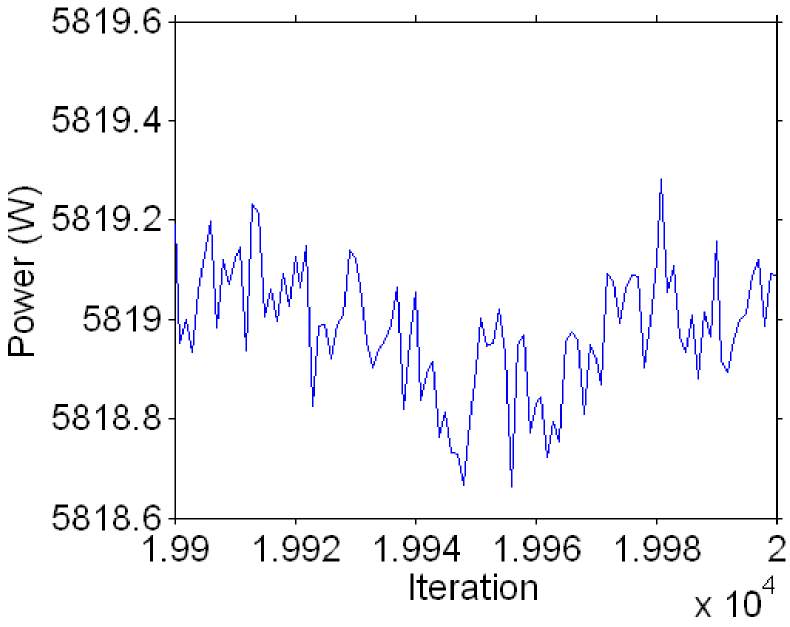
5-mm Square Aperture Conclusions

- Modeling results showed that the field could be decomposed well into Hermite-Gauss modes.
- The small aperture produced results consistent with that expected from Rigrod analysis within a reasonable area scaling factor.

Intensity Frame Averaging on the Full-Aperture (60-mm Square) Laser Model

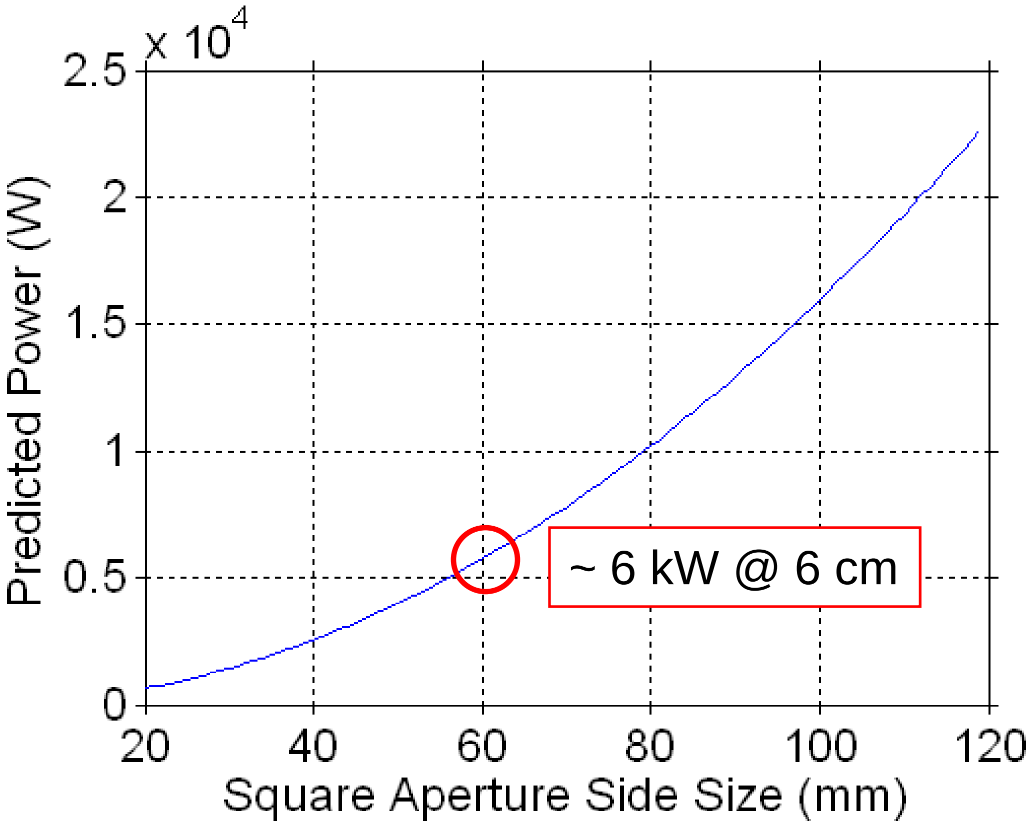
Full Aperture Power Output





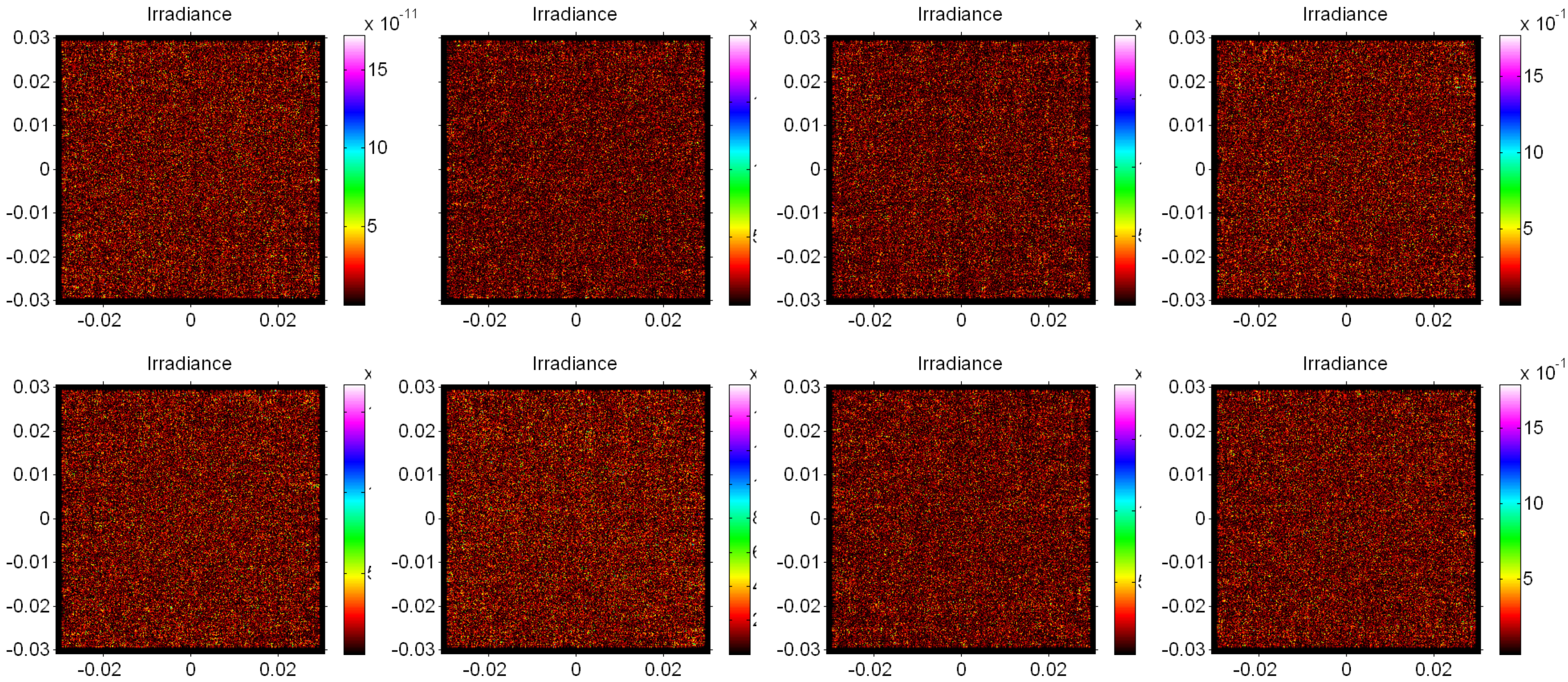
$P_{out} \sim 6$ kW

WaveTrain result very comparable with the Rigrod analysis.



- Rigrod analysis very accurate in the large aperture case because aperture is >> than TEM_{00} mode size.
- As we will see in the next slides, large number of modes results in “uniform” beam that gives better match to Rigrod assumptions.

Intensity profiles near the end of the simulation appear almost random because of the many modes lasing, but …



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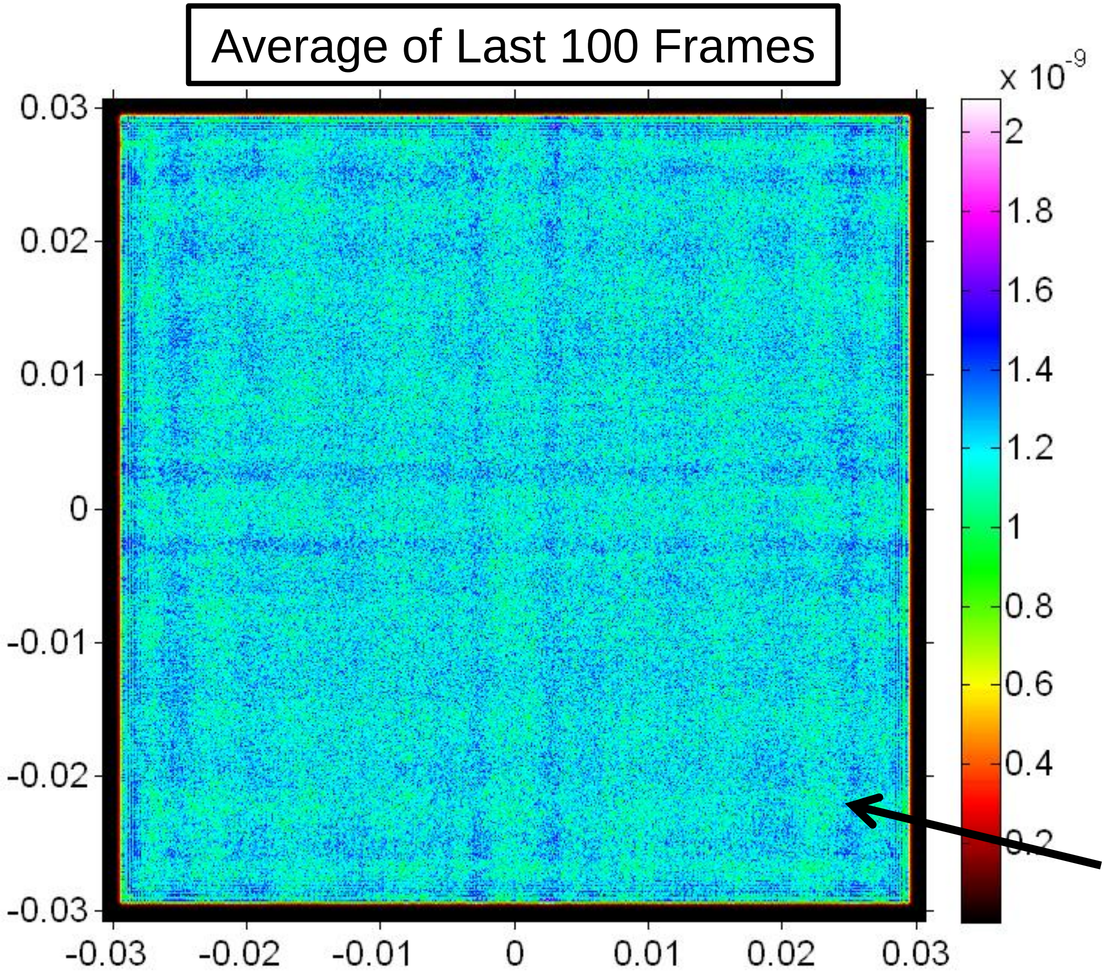
Averaging the last 100 frames shows more uniformity and some expected structure.



Beam profile essentially uniform as expected

Minor ripple probably due to aperture Fresnel ringing

Full-Aperture Conclusions

- Beam power level consistent with Rigrod analysis
- Beam profile consistent with expectations of a uniform beam
  - Large number of modes within aperture
  - Temporal mode phasing produces “average uniform” beam
- Power level and average beam profile appear accurate, but more experimental anchoring is needed

Conclusions

- The model of the stable resonator with gain using the internal intensity averaging appears to
  - converge much faster and
  - operate much more stably
- Results are consistent with theory
  - Power output consistent with Rigrod calculations

Future Work and Acknowledgements

- We need to perform more anchoring to experimental data to complete the verification of this new technique.
  - Experiment: Aperture in a small stable resonator.
- We need to consider how the different frequencies impact model performance.
- Acknowledgements
  - A. Paxton and A. E. Siegman for technical discussions
  - Funded via the LADERA contract

Questions?

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Backup Slides

RADICL Reduced-Aperture Studies

Reduced Aperture Study Goals

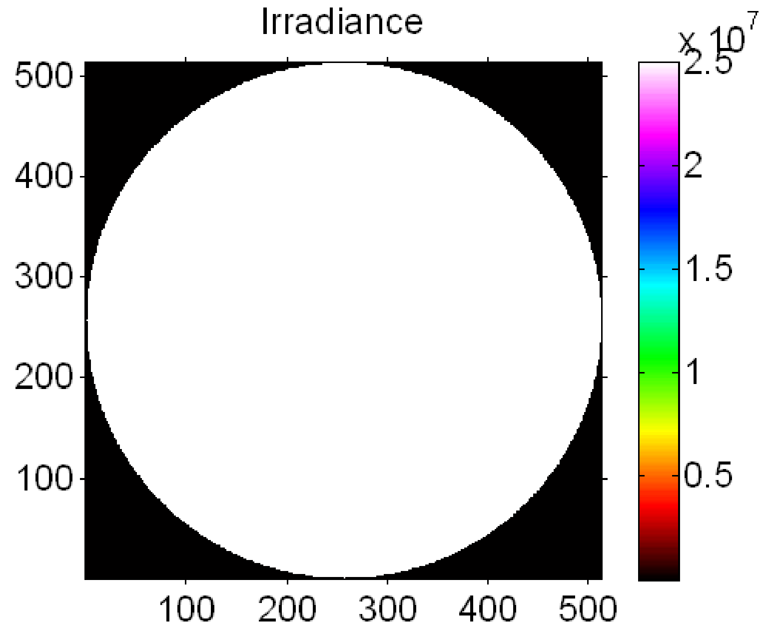
- Study a resonator design with significantly lower mesh point requirements
- Determine if results match theory
- If no theory exists, determine if results are consistent with expectations.

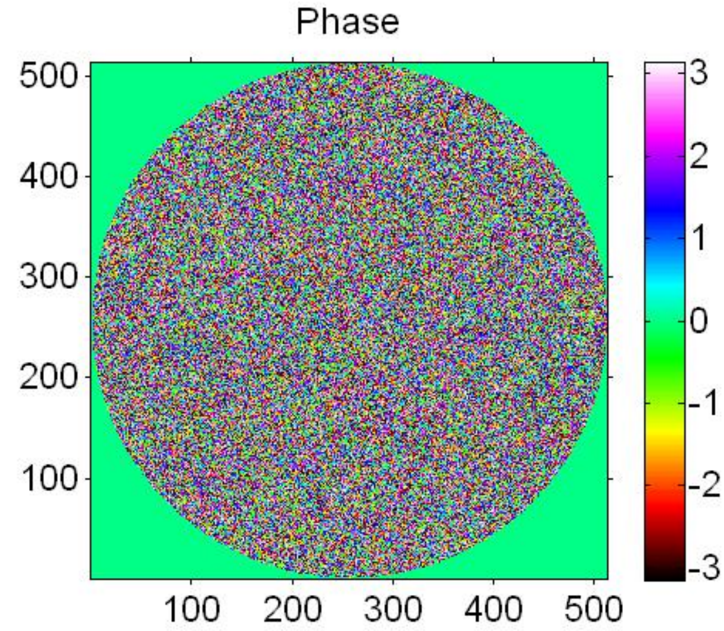
Reduced Aperture Model Parameters

Parameter	Value	Units
nxy	512	points
dxy	1.9531e-005	m/point
wavelength	1.315e-6	m
I_{sat}	67350000	W/m^2
aperture width	0.005	m
Rc	10	m
L	0.8	m
gSS	0.19435	gain / m
Gain length	0.254	m
Output coupling	0.05	fraction

Reduced Aperture Initial Field

- Bwomik method employed – uniform irradiance and random phase intended to seed all available resonator modes

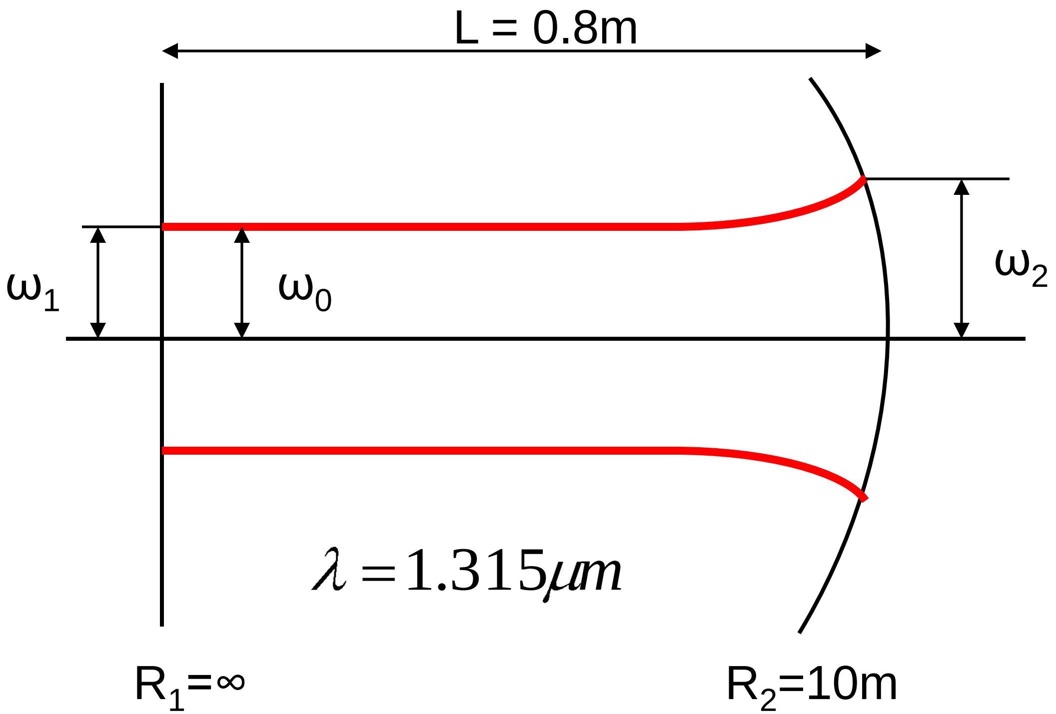




Comparison of the Basic WT Model Results to Beam Size Theory

Predicted Gaussian Radius

TEM00 Mode Radius for Plano Concave Resonator



$$\omega_1^2 = \omega_0^2 = \frac{\lambda}{\pi}\sqrt{L(R_2 - L)}$$

$$\omega_2^2 = \frac{\lambda}{\pi} R_2 \sqrt{\frac{L}{R_2 - L}}$$

$$\omega_0 = \omega_1 = 1.0656mm$$

$$\omega_2 = 1.11mm$$

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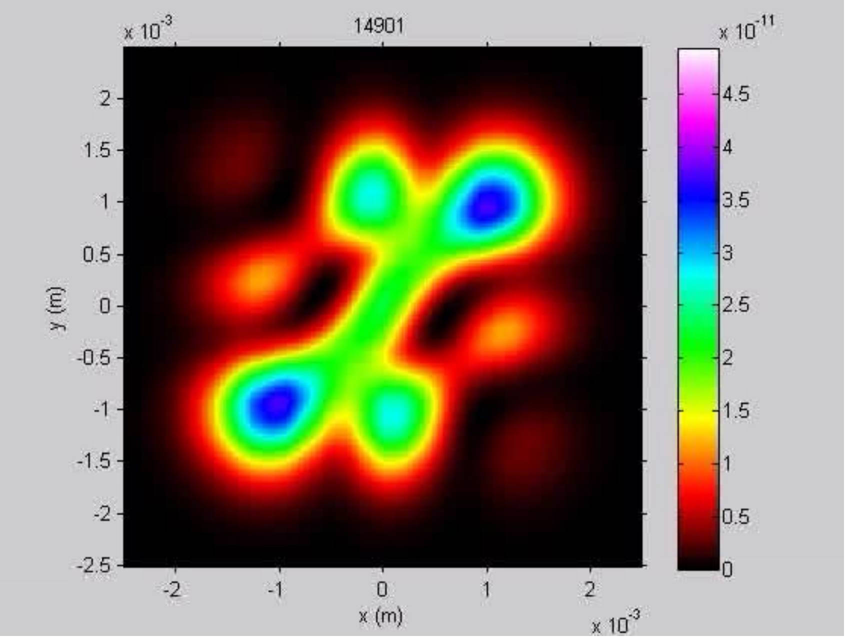
Reduced Aperture Beam Profile

- Movie shows last 100 irradiance frames
- Change in pattern over time is due to time-dependent phase differences between dominant HG modes of the cavity
- This time dependence is easily verified through a simple geometric ray analysis

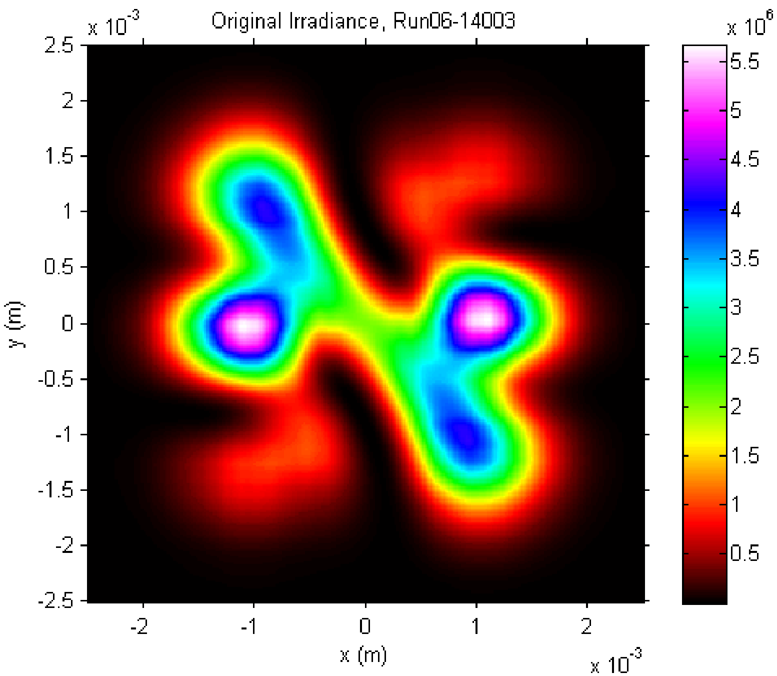


Aperture Diameter = 5mm

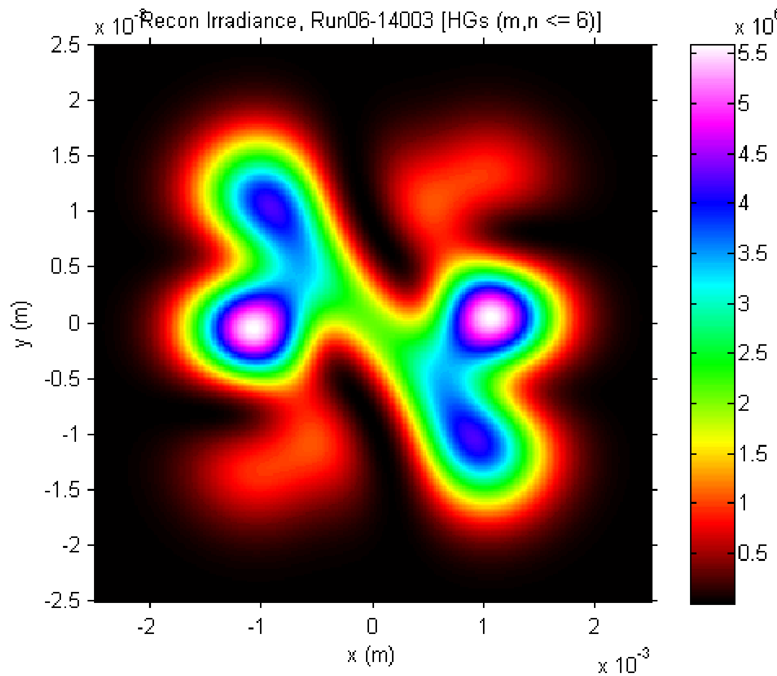
HG Decomposition – Accuracy

- Using theoretical waist, decompositions of the fields were performed, and show very good accuracy

Original Irradiance



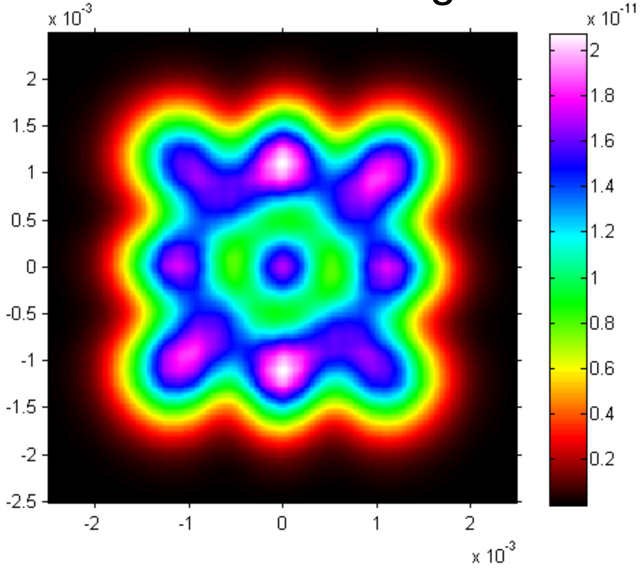
Reconstructed Irradiance



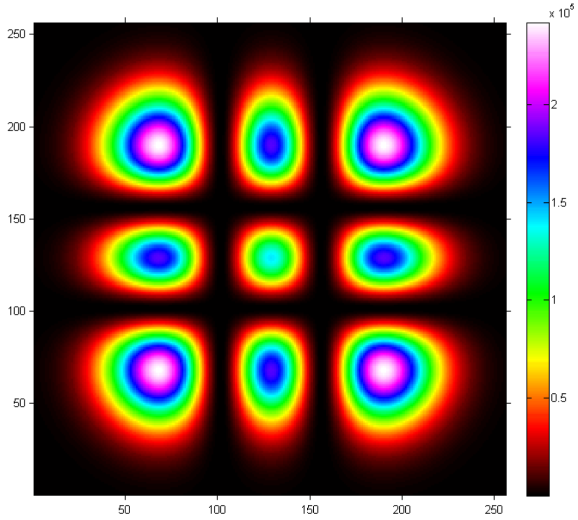
Beam Profile Decomposition

- In fact, the time-average of the last 100 irradiance frames has some characteristics of a HG22 mode:

100-Frame Avg.



HG22



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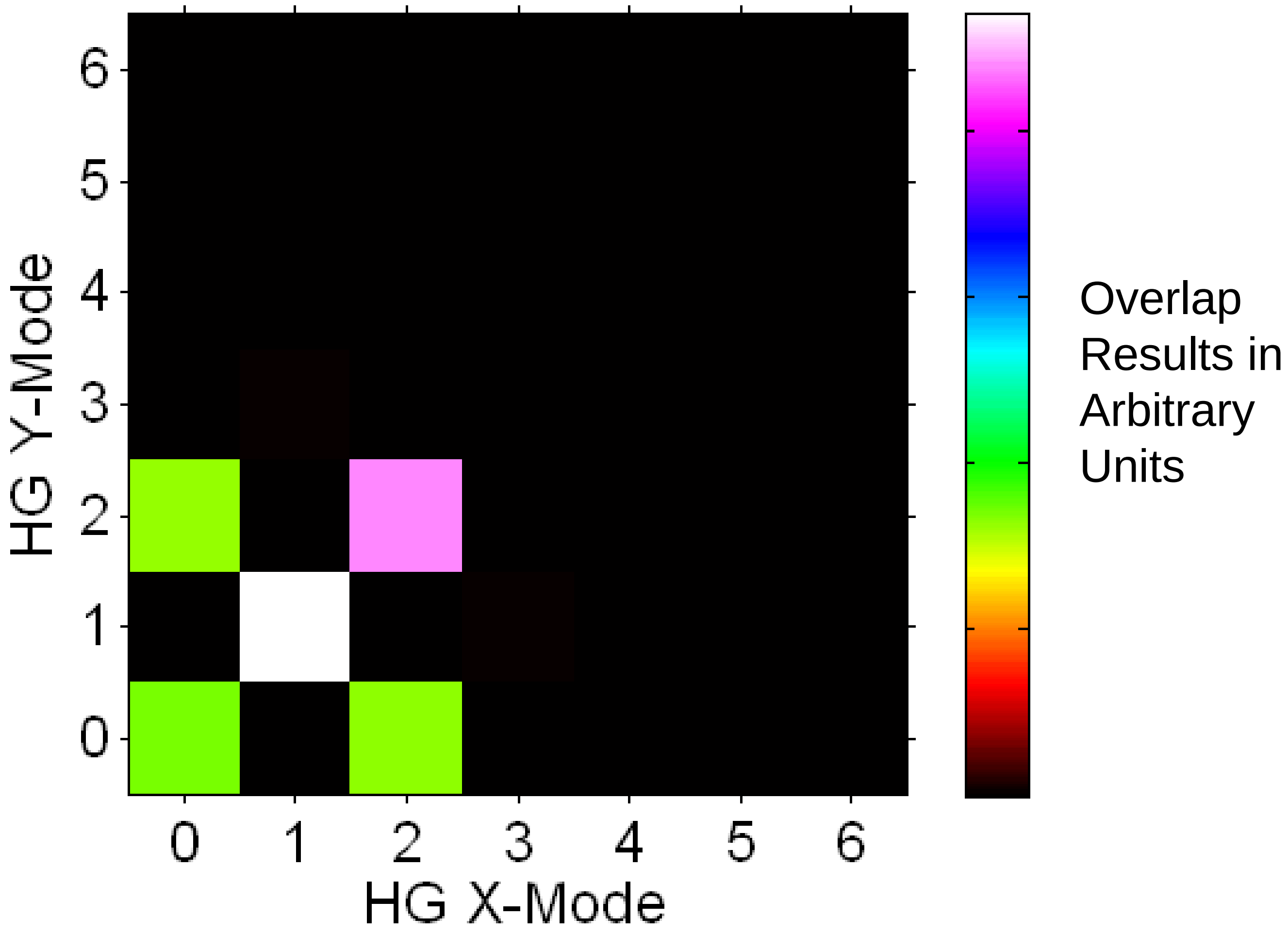
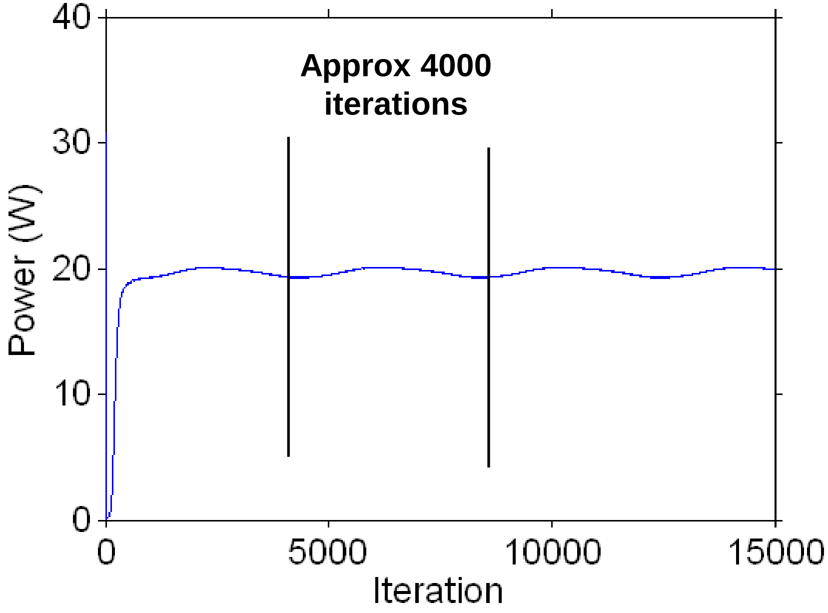
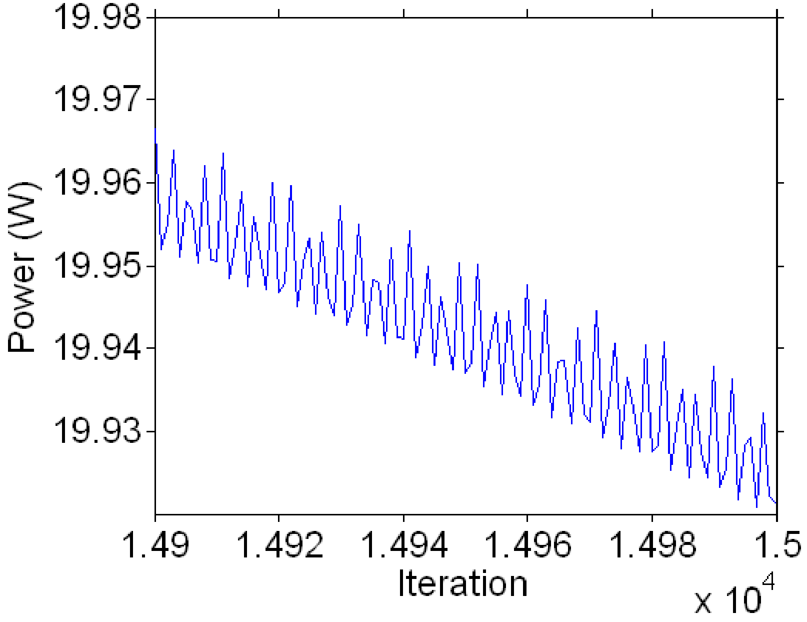
HG Decomposition



Reduced Aperture Power Output

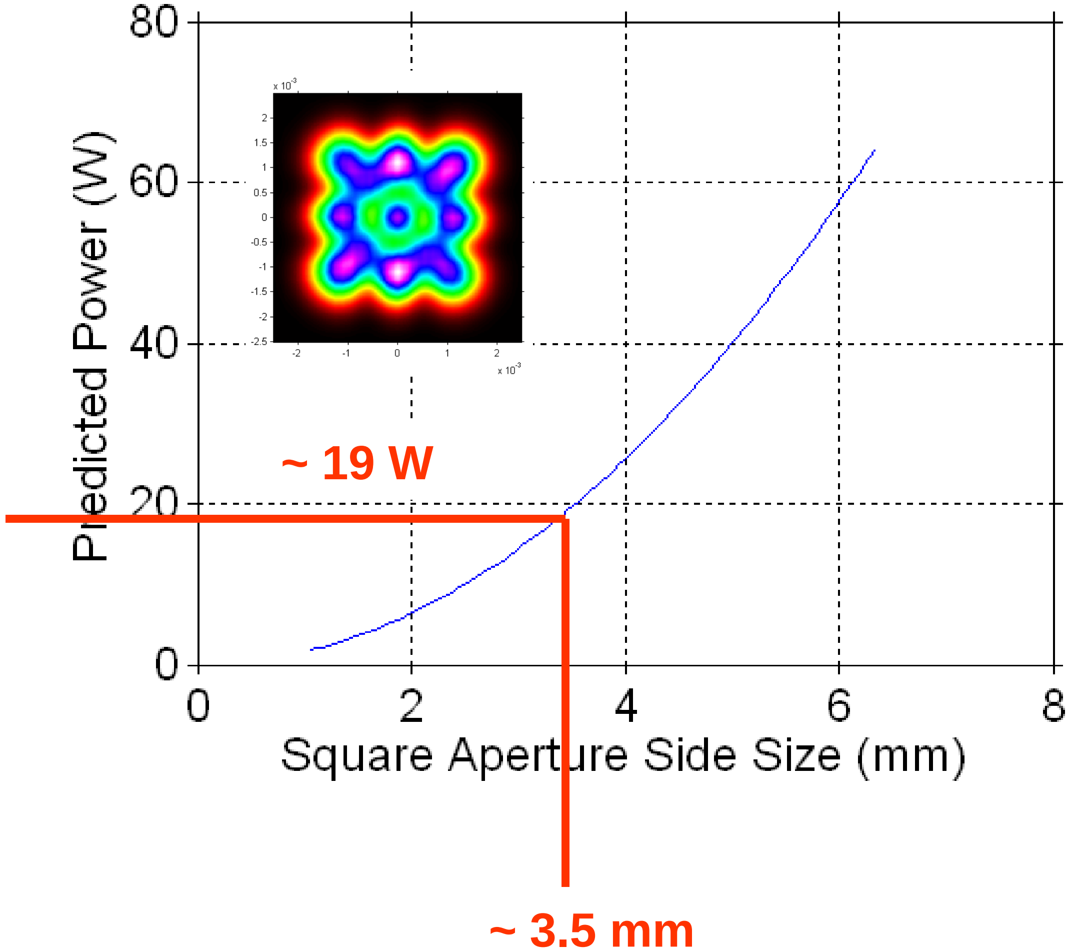




Slow oscillation observed over ~4000 iterations

Fast oscillation observed over ~2.75 iterations

Rigrod Analysis of Power Level



- Rigrod predicts power for "large output coupling" and for uniform field
- Thus we would expect a non-uniform field to have lower power (or equivalently a smaller "effective" aperture)
- I_Rigrod = 1.5974e+006 W/m^2
- Flattening simulated fields to I_Rigrod matches closely to 3.5 mm aperture

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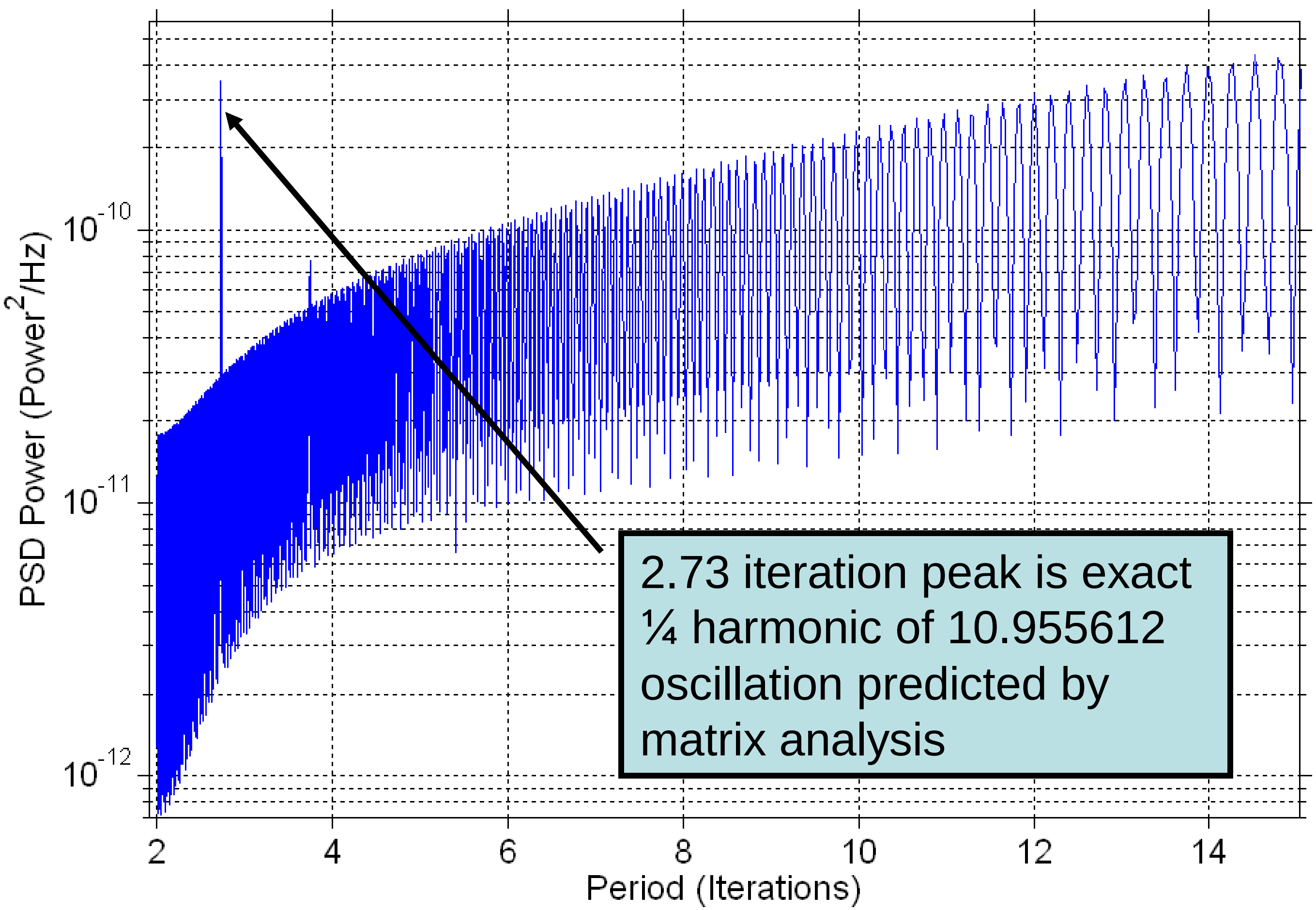
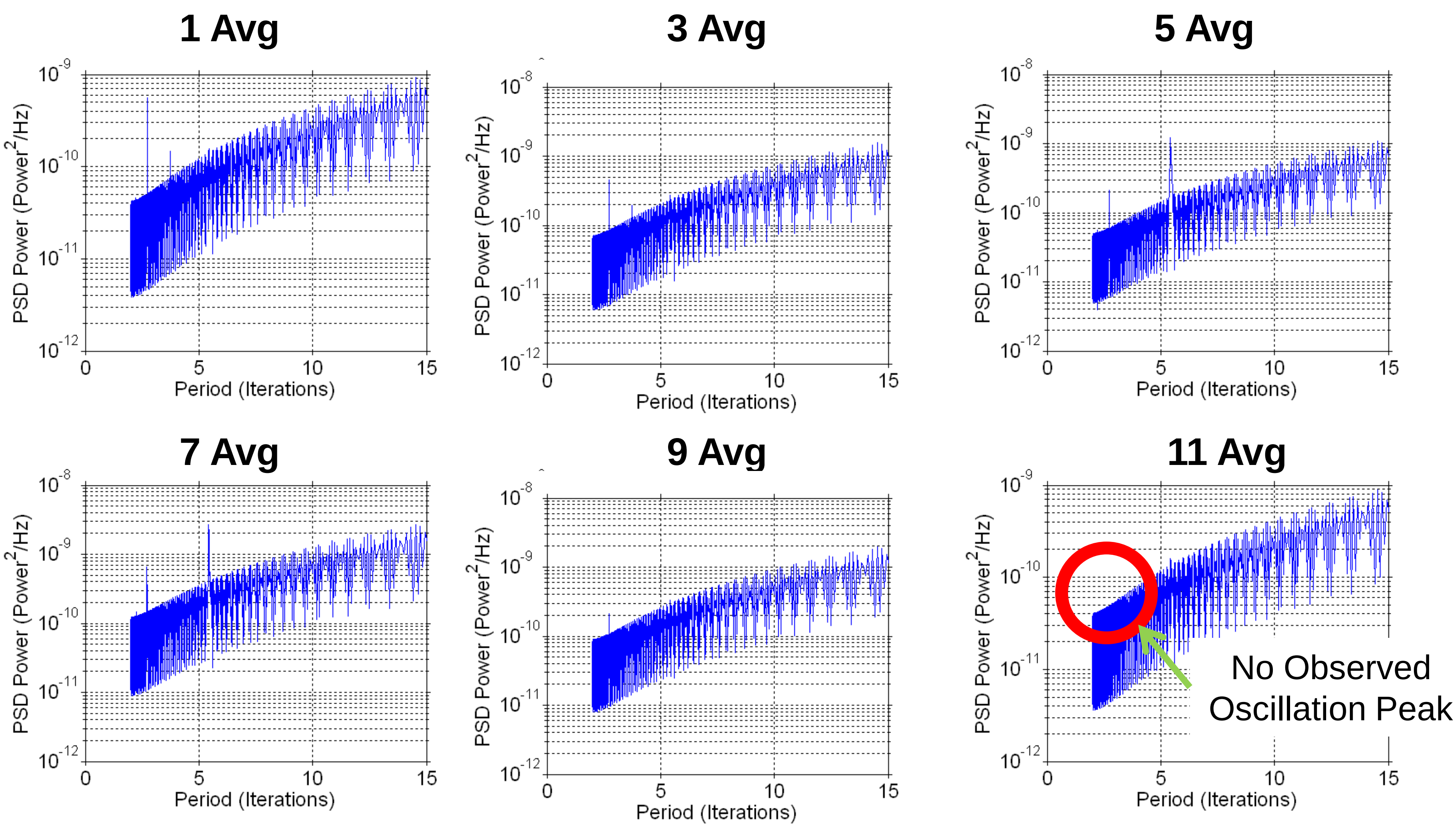
Spectrum in Region of Fast Oscillation



Implemented Intensity Frame Averaging for Gain Medium

- After observing the strong fast oscillations in the resonator output power, we realized that
  - they were likely caused by the intensity profile being printed into the simple saturable gain medium and
  - we might be able to suppress them by averaging the intensity over the number of round-trips to image.
- We implemented a component in WaveTrain that averaged the previous set of frames using a user-specified window.

Output Power PSD (vs. Period)



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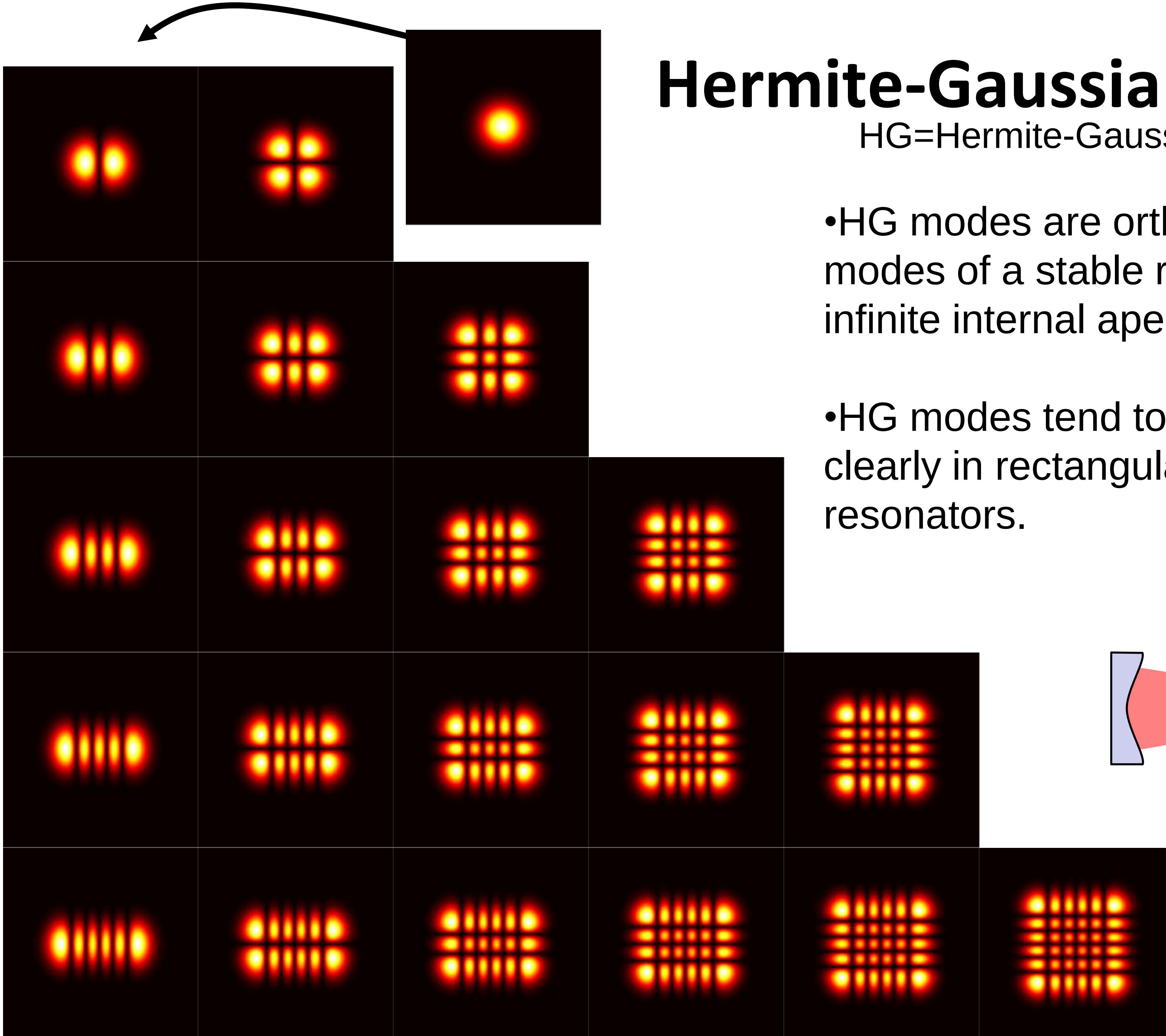
Reduced Aperture Conclusions - Summary

- The model predicted the lowest order mo size accurately.
- Output power is consistent with Rigrod theory if using appropriate compensation for beam shape.
- Intensity averaging of the input to the saturable gain over the number of round-trips to image successfully removes the high speed oscillation.
- Intensity averaged over the number of round-trips to resonate looks like the expected HG composition.

Modeling Conclusions

- We have introduced and demonstrated a technique for determining the reduced mesh for stable resonator modeling
- We have demonstrated a new technique for achieving a stable wave-optics modeling result of a multi-mode resonator by averaging the intensity over the number of round-trip passes required to image.

Backup Slides

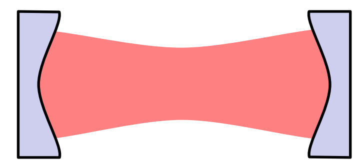


Hermite-Gaussian Modes

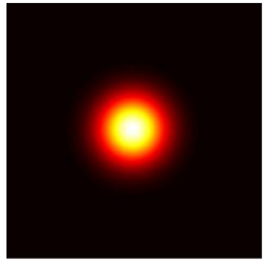
HG=Hermite-Gaussian

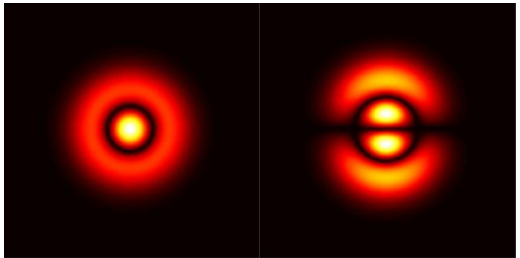
•HG modes are orthogonal modes of a stable resonator with infinite internal apertures.

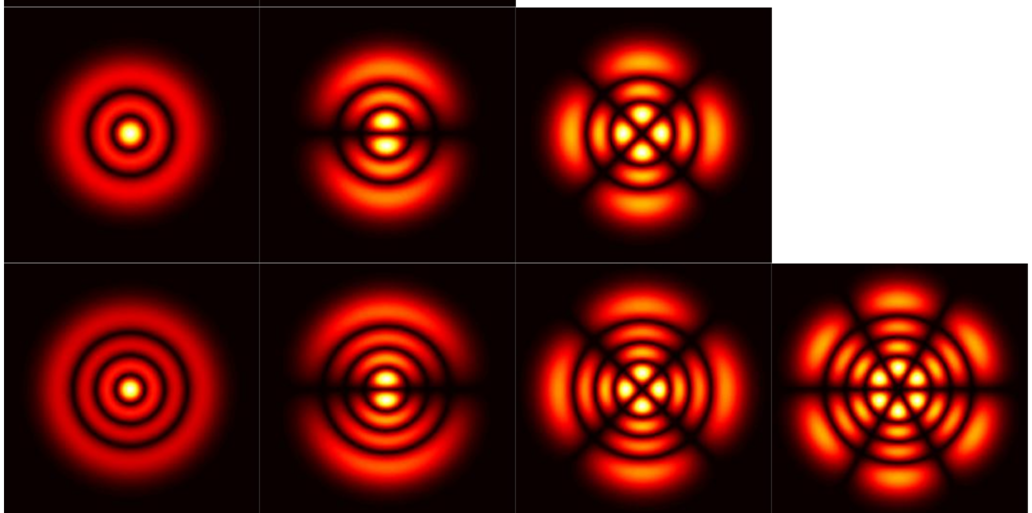
•HG modes tend to appear clearly in rectangular-aperture resonators.

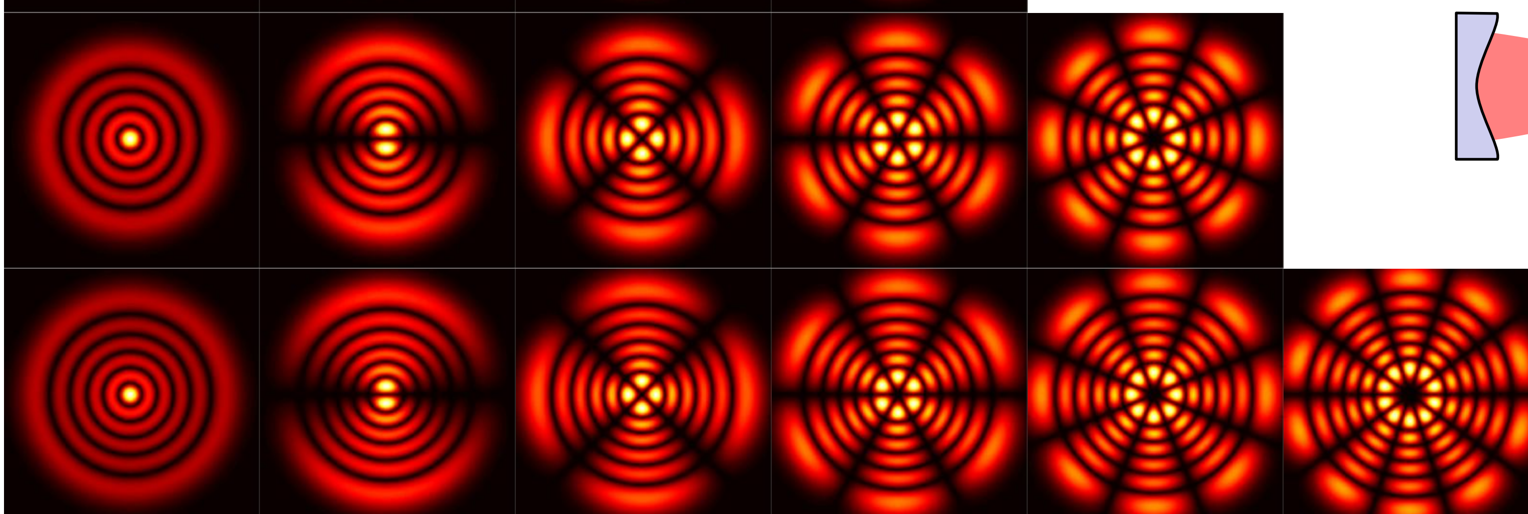


Laguerre-Gaussian Modes





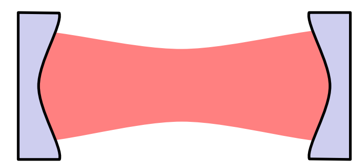




LG=Laguerre-Gaussian

- LG modes are orthogonal modes of a stable resonator with infinite internal apertures.

- LG modes tend to appear clearly in circular-aperture resonators.



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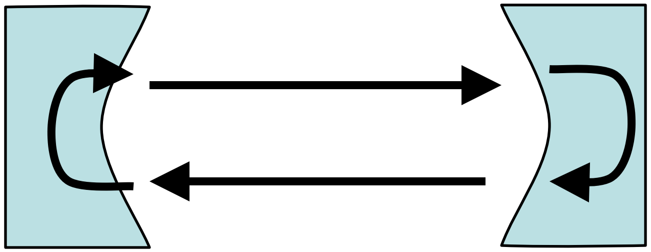
The Iterative Fourier Transform (aka Fox & Li) Technique

- A field is propagated through repeated round-trips until the field has converged to a stable field distribution.
- This is a commonly used technique for simplifying the 3D solution of Maxwell's Equations into a 2D problem (i.e. Gerchberg-Saxon technique)



A. G. Fox and T. Li. "Resonant modes in a maser interferometer", Bell Sys. Tech. J. 40, 453-58 (March 1961).

A. G. Fox and T. Li, "Computation of optical resonator modes by the method of resonance excitation", IEEE J. Quantum Electronics. QE-4, 460-65 (July 1968).

Comments on Fox & Li Solutions

- Solution is for an instantaneous state, which is typically the steady-state of the laser.
- Stable resonators are much more computationally intensive to model than unstable resonators.
  - We attribute this to the geometric output coupling of an unstable resonator.
  - Larger eigenvalue difference between fundamental mode and the next higher-order mode.
- Not generally appropriate for pulsed or time-varying solutions unless the time-varying nature is much slower than a resonator round-trip time.
  - This is analogous to a split-time modeling techniques.